ELECTRIC TRAINS

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ELECTRIC TRAINS

ELECTRIC TRAINS

BY

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"HEAVY ELECTRICAL ENGINEERING," "ELECTRICITY," "ELECTRIC MOTORS,"
AND "DYNAMO DESIGN"

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PREFACE

It is indicative of the rapid progress of the application of electricity to the propulsion of railway trains, that it is no longer possible, within the bounds of a single volume, to cover the entire subject with any approach to adequacy. Indeed, in limiting my programme to Electric Trains, I have still found the field too wide to be dealt with effectively, and I have consequently confined the treatment to Electric Trains for City and Suburban Service, thus purposely excluding the large and important subject of Electric Locomotives. Even with this restricted programme I have considered that it is impossible, without incurring the risk of diverting the reader's attention from the logical development of my subject, to introduce much material descriptive of the apparatus comprised in the electrical equipment. I have endeavoured to make amends for this by including references to publications in which useful descriptive matter may be found.

There is a dearth of published matter where a reader can find set forth coherent and simple descriptions of thoroughly practical methods of calculating the required energy for operating electric railways. I do not know of any work other than the present book in which the approximate estimation of the cost of the rolling stock of electrically-propelled motor-coaches and trains, proportioned for a given service, is worked out rationally and with the certainty of arriving at reliable results for the comparative purposes of preliminary estimates. Most of the interesting and valuable essays in these directions are too ponderous, and their authors have been hampered by the involved and cumbersome methods which they have employed. Although portions of these essays are of great value to those interested in electric railways, they are usually seriously overweighted with digressions of none too relevant a nature, and the methods set forth are needlessly complex.

The problems involved in railway electrification work are, in their broad aspects, not amenable to useful solution by other than more or less empirical methods based on rough, practical tests. After the broad solution has been reached, and after the main features of the scheme have been laid down, *then*, and *then only*, is the stage reached when close detailed work on the many component points involved should be undertaken.

The electrical engineer will be out of his element in designing trucks and car bodies, and he will be well advised, even in the matter of the suitable location of the electrical apparatus, to confine his part to that of simply placing at the disposal of experienced steam-railway engineers his special knowledge of electrical apparatus and methods. Holding this belief, I have not included in this treatise matters relating to details of the design of trucks and car bodies. I consider that it would be inappropriate, at any rate in a book of the small compass of the present treatise, to include information on these points.

I am hopeful that not only electrical engineers and students, but also railway engineers, will find the present treatise of assistance in making clear the most pertinent considerations governing the electrical aspects of the design and operation of Electric Trains for City and Suburban Service, and with a view to the achievement of this purpose, I have not considered it expedient to apply the space at my disposal to other than the most distinctly relevant portions of the subject.

I have taken the mile as the unit of length. It is, however, characteristic of electrical engineering, that calculations are expedited by the use of the metric system. Consequently for *short* lengths I have employed the meter, of which there are 1609 in the mile. Accelerations are given in miles per hour per second, and I have employed the abbreviation "ml phps," since the abbreviation "m" is reserved for the meter. With the desire to avoid bigotry in this matter of units, I have, in such cases as the dimensions of rolling stock, freely employed feet and inches. The British ton and the metric ton differ from one another by only 1·8 per cent., and since this variation is of absolutely no consequence in such a subject as that dealt with in the present treatise, I have regarded the British ton and the metric ton as identical. Engineers should keep in mind that in the

case of many descriptions of electric railway work carried out in America, the authors of the British papers and books republishing these descriptions have often not taken the trouble to translate the weights from the 2000-lb ton used in America to the British or metric ton used everywhere else. There are many instances in which it would appear that the authors of these papers have not noticed that another than the British ton is employed. In view of the complacency with which this 11 per cent. indefiniteness is assimilated by the engineering community, I see no reason to anticipate criticism of my plan of ignoring the 1·8 per cent. difference between the British and metric ton, and I consider it a great advantage that in this matter of the ton, we have a unit common to both the British and the metric systems. It should not be difficult to take the next step, and to employ the one-thousandth part of the ton as the smaller unit of weight. Whether this be described as 0·001 ton or as 1 kg is immaterial. In this book I have employed the kilogram as a convenient designation for the thousandth part of the ton. Temperatures are in all cases given in the Centigrade scale.

In building up a basis for my methods, I have perused with much profit Aspinall's papers ("Proc., I.C.E.," vol. cxlvii. p. 241, and "Proc., I.M.E.," 1909, No. 2, p. 423), Carter's contributions ("Journal, I.E.E.," vol. xxxvi. p. 231. Rugby Eng. Society, Feb. 18, 1909), and papers by Armstrong, Potter, Hutchinson and others, published in various volumes of the "Transactions of the A.I.E.E." While the reader will find in the text numerous reference to descriptions of apparatus, I cannot refrain from specially calling attention to a treatise entitled "Electric Traction," by Wilson and Lydall, since with respect to this special feature of descriptions of apparatus, the book is, in my opinion, unexcelled. "Electric Railway Engineering," in which Mr. H. F. Parshall and the present author collaborated, and Mr. Philip Dawson's "Electric Traction on Railways," also contain a great deal of detailed information concerning many notable cases of electrically operated railways. The present treatise is not to be considered as an alternative of any of the above-mentioned books, but as an attempt to accomplish the specific task indicated in this preface.

I wish to take this opportunity of expressing my thanks to Mr. J. R. Chapman, Chief Engineer of the Underground Electric

Railways Co., of London; to Mr. R. P. Brousson, General Manager of the Great Northern and City Railway; to Mr. J. A. F. Aspinall, General Manager of the Lancashire and Yorkshire Railway; to Mr. H. F. Parshall, Consulting Engineer to the Central London Railway; to Mr. E. P. Grove, Chief Engineer of the Central London Railway; to Mr. C. H. Merz, Consulting Engineer to the North Eastern Railway; and to Messrs. Dick, Kerr and Co., and Messrs. The Siemens-Schuckert Dynamo Works, for their courtesies in providing me with data of their undertakings.

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LIST OF ABBREVIATIONS EMPLOYED.

Foot, feet	ft
Feet per second per second	ft psps
Horse-power	hp
Inch	in
Kilogram	kg
Kilogram-meter	kg m
Kilogram-meters per second	kg m ps
Kilometer	km
Kilometers per hour	km ph
Kilowatt	kw
Kilowatt-hour—Kelvin	kw hr
Meter	m
Meters per second	m ps
Meters per second per second	m psps
Miles per hour	ml ph
Miles per hour per second	ml phps
Watt-hour	w hr
Watt-second	w sec

ELECTRIC TRAINS

CHAPTER I

SPEED-TIME DIAGRAMS

In making calculations relating to the electric propulsion of trains, one of the first steps consists in constructing "speed-time" diagrams.* A speed-time diagram is usually drawn with times, in seconds, from the instant of starting the train, as abscissae, and with the corresponding speeds, in miles per hour, as ordinates. Let us take the case of a passenger train operating under the average conditions which obtain

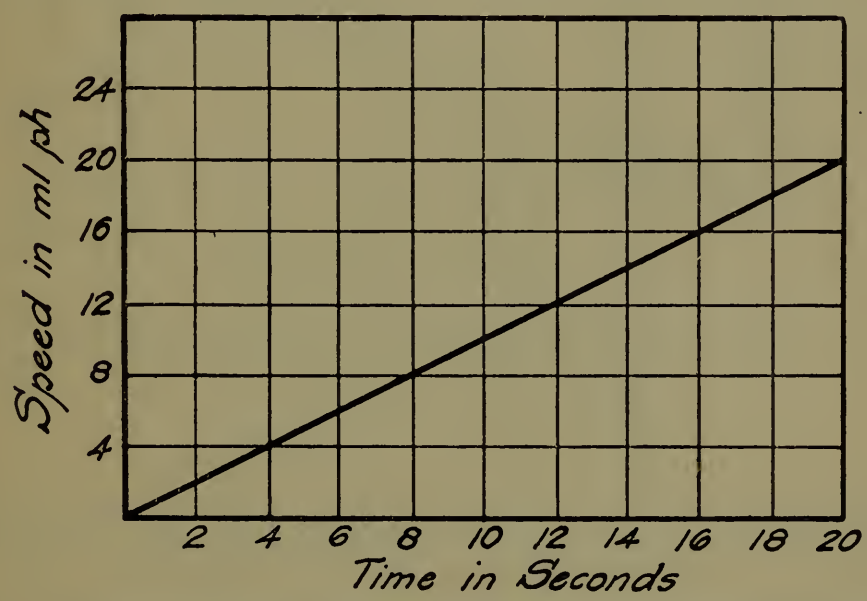

Fig. 1.—Accelerating Portion of Speed-time Diagram, with Acceleration maintained constant at one ml phps.

on such lines as those of the Underground Electric Railways Co. of London, or of the Central London Railway. At the instant of switching on the electricity, *i.e.* at the instant of starting, the speed is zero.

* An excellent mathematical treatment of the subject of speed-time diagrams is given by C. O. Mailloux, in a paper entitled "Notes on the Plotting of Speed-Time Curves," and read before the American Institute of Electrical Engineers ("Transactions," vol. xix. p. 901).

One second later the speed may be one mile per hour (one ml ph) or more. If, at the end of the first second, the train has acquired a speed of one ml ph, the average acceleration during the first second is said to have been one mile per hour per second (one ml phps). If this acceleration is maintained uniform for 20 seconds, the speed at the end of the 20th second will be 20 ml ph. For such a case the first part of the speed-time diagram is drawn as shown in Fig. 1.

The acceleration will, in practice, vary from instant to instant.

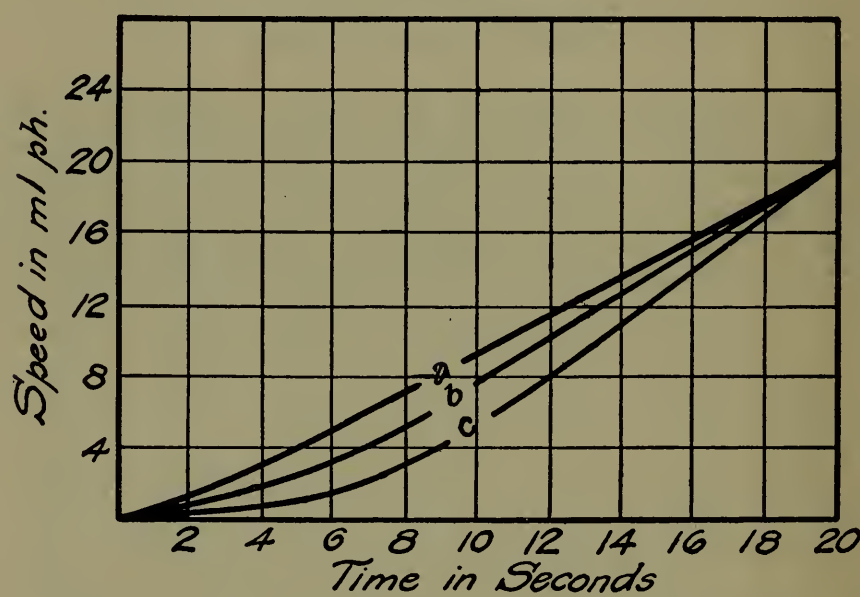

Fig. 2.—Accelerating Portions of Speed-time Diagram, showing how the Acceleration may be gradually increased up to its Final Value.

Curve *a*, Final Acceleration 1·1 ml phps.
" *b*, " " 1·2 "
" *c*, " " 1·5 "

The train will usually start off with only comparatively moderate acceleration, otherwise discomfort would be experienced by the passengers. But by the end of a very few seconds such a train as that instanced will be accelerating at one ml phps, or thereabouts, and the maximum acceleration may be 1·5 ml phps, or even more. Thus the first part of the speed-time diagram is usually more like one of the curves in Fig. 2, which represent accelerations gradually increasing to 1·1, 1·2, and 1·5 ml phps for curves *a*, *b*, and *c* respectively.

The corresponding *instantaneous* values of the acceleration are shown in Fig. 3. Curve *d* corresponds to the constant acceleration of Fig. 1, and curves *a*, *b*, and *c* correspond to the gradually increasing accelerations of curves *a*, *b*, and *c* respectively of Fig. 2.

In all four cases, the speed at the end of the 20th second is 20 ml ph, the average acceleration being therefore, in each case, 1·0 ml phps. But, obviously, the distance covered in the 20 seconds is less in the last three cases than in the first case. In the first case (*i.e.*

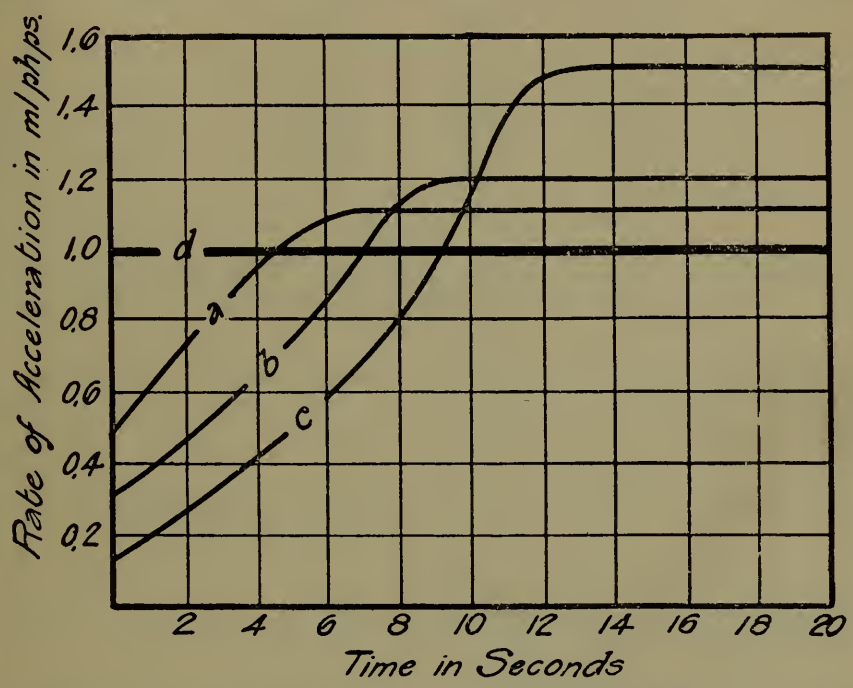

FIG. 3.—Curves showing the Variation of the Acceleration with Time, corresponding to the Curves in Figs. 1 and 2.

Fig. 1, and curve *d* of Fig. 3), the *average* speed during the 20 seconds is 10 ml ph, and the distance covered in the 20 seconds is—

$$\frac{20}{3600} \times 10 = 0{\cdot}0555 \text{ mile} = 89{\cdot}2 \text{ m.*}$$

In the case of curve *a* in Fig. 2, the average speed is 9·5 ml ph, and the distance covered is only—

$$\frac{20}{3600} \times 9{\cdot}5 = 0{\cdot}0528 \text{ mile} = 84{\cdot}7 \text{ m.}$$

In the four cases, the initial and final acceleration, the mean speed, and the distance covered during the first 20 seconds are set forth in Table I.

---

* 1 mile = 1609 meters (1609 m).

The reader will find that the most practicable course, pending the general introduction of the kilometer (km) as a substitute for the mile, is to express all considerable distances in miles, and all short distances in meters. While the meter is already extensively used, and is practically as familiar as the foot or the yard, it will take many years to supersede the mile by the kilometer.

TABLE I.—DATA OBTAINED FROM THE CURVES OF FIGS. 1 TO 3.

Curve of acceleration (of Fig. 3).	Acceleration (ml phps).			Mean speed (ml ph).	Distance covered (m).
	Initial.	Final.	Mean.		
d . .	1·00	1·00	1·00	10·0	89·2
a . .	0·50	1·10	1·00	9·5	84·7
b . .	0·31	1·20	1·00	8·3	74·1
c . .	0·14	1·50	1·00	7·0	62·5

The distances covered during the first 20 seconds are plotted as a function of the initial accelerations, in the curve of Fig. 4. From

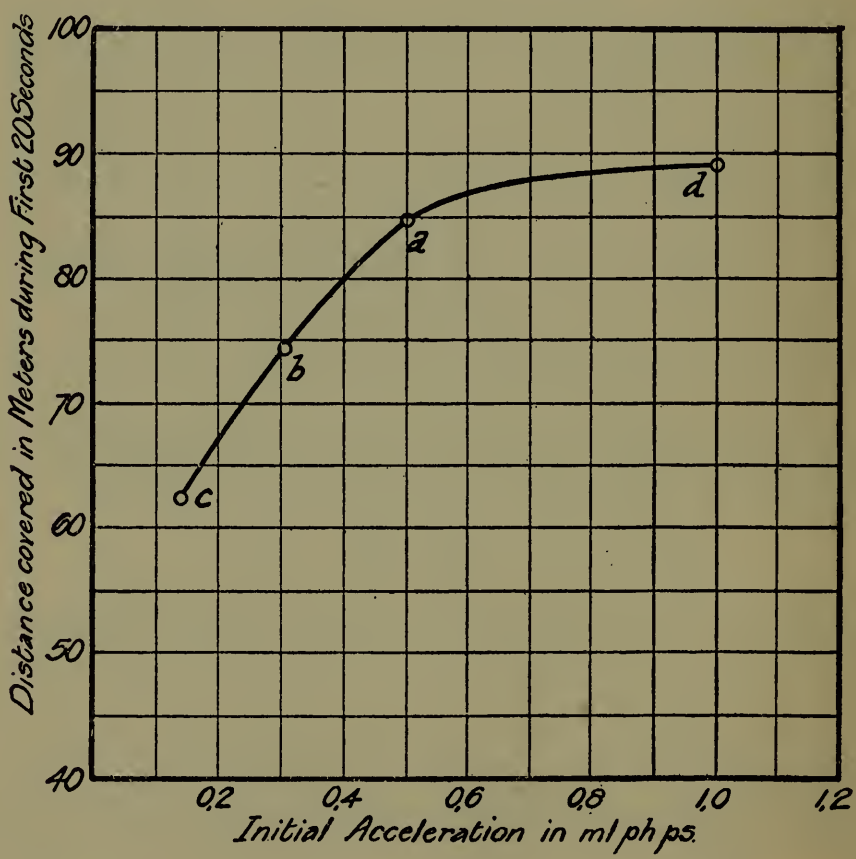

FIG. 4.—Curve showing Distance travelled for the first 20 seconds of Run with the Various Initial Accelerations of the Curves of Fig. 3.

this curve it will be seen that, so far as is consistent with other considerations, it is very important that the initial acceleration shall be fairly high, in order that the train may perform its journey in the

shortest practicable time. The considerations standing in the way of a high initial acceleration are :—

1. The Stress imposed on the Rolling Stock.—Thus the higher the acceleration employed, the stronger and heavier must be the rolling stock.

2. The Comfort of the Passengers.—If, at the very first instant of starting, the acceleration is moderate, it can, after that, be rapidly increased to even 2 ml phps without undue discomfort to passengers, provided the alteration in the acceleration is accomplished smoothly, *i.e.* uniformly. Consequently, consideration has to be given to the **rate of accelerating the acceleration.** But this is a refinement which, while it must be carefully kept in mind, need rarely concern us in preliminary calculations.

3. The Effects on the Power House, the Line, and the Sub-Stations.—The higher the acceleration, the greater will be the instantaneous peaks of load on the system. With a large number of trains running on short headway, the peaks tend to overlap, and are consequently less noticeable, but with infrequent trains, these peaks of load constitute a serious objection to employing a high acceleration.

4. The Effect on the Electrical Equipment on the Train.—The greater the acceleration, the greater is the instantaneous load on the train equipment, and the more severe are the conditions to which such parts as the commutators and brushes of the motors, and the contacts of the controllers or contactors, are subjected. Consequently, the employment of very high accelerations may in some instances necessitate heavier and more expensive electrical equipments than would otherwise be required.

Notwithstanding these considerations, maximum accelerations well above one ml phps are commercially employed in the electrical operation of such trains as those on London's underground railways, as against accelerations of the order of 0·3 to 0·4 ml phps for equivalent *steam* trains. Mr. Mordey has analysed the conditions of operation of the electric trains on the Liverpool Overhead Railway, and has shown that on certain occasions the acceleration, 2 or 3 seconds after starting, reached 2·8 ml phps.* In some tests, made in America by the General Electric Co. with a 65-ton train, the average acceleration during the first 5 seconds worked out at nearly 3 ml phps.† But even so relatively low an acceleration as 1·5 ml phps has been criticised ‡ as involving expensive and heavy rolling

* "Proceedings, Institution of Civil Engineers," February, 1902.
† "Transactions, American Institute of Electrical Engineers," vol. xix. p. 844.
‡ Mr. Roger Smith, in *Railway Gazette*, February 5, 1909, p. 170.

stock construction to withstand the attendant stresses. Taking it all in all, however, it may be said that electrically-propelled passenger trains on well-built railways will usually be operated with a maximum of commercial advantage, when the average acceleration, during at least the first 10 seconds from starting, is from 0·6 to 1·5 ml phps. For reasons which will be better understood at a later stage, the lower of these accelerations will be approached when the distance between stops is greater and the schedule speed lower, whereas, for short distances between stops, high schedule speeds are only possible when the higher of these accelerations is approached. The customary accelerations of steam-hauled passenger trains range from 0·2 to 0·4 ml phps.*

On the Continent, train speeds are expressed in km ph (kilometers per hour), and accelerations in m psps (meters per second per second). In Great Britain, train speeds are expressed in ml ph, and accelerations are expressed either in ml phps (miles per hour per second), or in ft psps (feet per second per second). The use, sometimes of the one and sometimes of the other of these last two units, when expressing accelerations, is troublesome, but it seems unlikely that any uniformity will be arrived at in the near future. It is beyond all question more convenient for practical railway calculations, to express accelerations in ml phps, since this leads to great convenience in calculating. Thus, if a train has accelerated at the average rate of 1·2 ml phps for 9 seconds, its speed at the end of the 9th second is—

$$9 \times 1{\cdot}2 = 10{\cdot}8 \text{ ml ph.}$$

In Table II. are given, for various units, equivalent values of speeds and accelerations.

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* "Comparative Acceleration Tests with Steam Locomotive and Electric Motor-cars," by B. J. Arnold and W. B. Potter. "Transactions of the American Institute of Electrical Engineers," vol. xix. p. 833.

TABLE II.—CONVERSION TABLE FOR SPEED AND ACCELERATION.

Miles per hour (ml ph).	Feet per second. (ft ps).	Kilometers per hour (km ph).	Meters per second (m ps).	Speed.	Miles per hour (ml ph).	Feet per second (ft ps).	Kilometers per hour (km ph).	Meters per second (m ps).
Miles per hour per sec. (ml phps).	Feet per sec per sec. (ft psps).	Kilometers per hr per sec. (km phps).	Meters per sec per sec. (m psps).	Acceleration.	Miles per hour per sec. (ml phps).	Feet per sec per sec. (ft psps).	Kilometers per hr per sec. (km phps).	Meters per sec per sec. (m psps).
0·1	0·1467	0·1609	0·0447		10	14·67	16·09	4·470
0·1364	0·2	0·2194	0·0610		10·23	15	16·46	4·572
0·2	0·2934	0·3218	0·0894		12	17·60	19·31	5·36
0·2728	0·4	0·4388	0·1219		12·42	18·22	20	5·55
0·3	0·440	0·4827	0·1341		13·64	20	21·94	6·10
0·4	0·5868	0·6436	0·1788		14	20·53	22·53	6·26
0·4091	0·6	0·6584	0·1829		15·53	22·78	25	6·94
0·5	0·7335	0·8045	0·2235		16	23·47	25·75	7·15
0·5454	0·8	0·8778	0·2438		17·05	25	27·43	7·62
0·6	0·880	0·9654	0·2682		18	26·40	28·97	8·05
0·6214	0·9114	1·0	0·2778		18·63	27·34	30	8·33
0·6818	1·0	1·097	0·3050		20	29·33	32·19	8·94
0·7	1·027	1·126	0·3129		20·46	30	32·91	9·14
0·8	1·172	1·287	0·3576		21·75	31·90	35	9·72
0·8182	1·2	1·316	0·3658		22	32·27	35·40	9·83
0·9	1·320	1·448	0·4023		23·86	35	38·40	10·67
0·9545	1·4	1·536	0·4267		24	35·2	38·60	10·73
1·0	1·467	1·609	0·447		24·85	36·45	40	11·10
1·023	1·5	1·646	0·457		26	38·13	41·84	11·62
1·242	1·822	2·0	0·556		27·28	40	43·88	12·19
1·25	1·833	2·012	0·559		27·96	41·01	45	12·50
1·364	2·0	2·194	0·610		28	41·06	45·06	12·51
1·5	2·20	2·414	0·671		30	44·0	48·28	13·41
1·705	2·5	2·743	0·762		30·68	45	49·38	13·72
1·75	2·567	2·816	0·782		31·06	45·56	50	13·88
1·863	2·733	3·0	0·833		32	46·93	51·50	14·30
2·0	2·930	3·219	0·894		34	49·87	54·72	15·20
2·046	3·0	3·291	0·914		34·09	50	54·86	15·24
2·25	3·30	3·622	1·006		34·18	50·12	55	15·27
2·386	3·5	3·840	1·067		36	52·80	57·94	16·09
2·484	3·644	4·0	1·111		37·27	54·67	60	16·66
2·5	3·668	4·023	1·118		37·5	55	60·35	16·76
2·728	4·0	4·388	1·219		38	55·73	61·15	16·99
2·75	4·034	4·426	1·229		40	58·67	64·37	17·88
3·0	4·40	4·828	1·341		40·40	59·24	65	18·06
3·068	4·5	4·938	1·372		40·91	60	65·84	18·29
3·107	4·557	5·0	1·389		42	61·60	67·59	18·78
3·25	4·767	5·230	1·453		43·49	63·79	70	19·43
3·409	5·0	5·486	1·524		44	64·53	70·81	19·67
3·5	5·13	5·633	1·565		44·32	65	71·32	19·81
3·728	5·47	6·0	1·667		46	67·47	74·03	20·56
3·75	5·5	6·035	1·676		46·61	68·35	75	20·83
4·0	5·87	6·437	1·788		47·73	70	76·81	21·34
6	8·8	9·656	2·682		48	70·40	77·25	21·46
6·21	9·11	10	2·778		49·6	72·90	80	22·21
6·82	10	10·97	3·048		50	73·33	80·47	22·35
8	11·73	12·87	3·576		51·14	75	82·30	22·86
9·32	13·67	15	4·167		52	76·27	83·68	23·25

TABLE II.—CONVERSION TABLE FOR SPEED AND ACCELERATION—*continued.*

Miles per hour (ml ph).	Feet per second (ft ps).	Kilometers per hour (km ph).	Meters per second (m ps).	Speed.	Miles per hour (ml ph).	Feet per second (ft ps).	Kilometers per hour (km ph).	Meters per second (m ps).
Miles per hour per sec. (ml phps).	Feet per sec per sec. (ft psps).	Kilometers per hr per sec. (km phps).	Meters per sec per sec. (m psps).	Acceleration.	Miles per hour per sec. (ml phps).	Feet per sec per sec. (ft psps).	Kilometers per hr per sec. (km phps).	Meters per sec per sec. (m psps).
52·82	77·47	85	23·50		90	132·0	144·8	40·23
54	79·20	86·90	24·14		90·2	132·1	145	40·28
54·55	80	87·78	24·38		92	134·9	148·0	41·13
55·61	82·01	90	24·99		92·05	135	148·1	41·15
56	82·13	90·12	25·03		93·18	136·7	150	41·64
57·95	85	93·27	25·91		94	137·9	151·3	42·02
58	85·07	93·34	25·93		95·45	140	153·6	42·67
59·03	86·58	95	26·39		96	140·8	154·5	42·91
60	88·0	96·56	26·82		96·3	141·3	155	43·06
61·36	90	98·76	27·43		98	143·7	157·7	43·81
62	90·93	99·78	27·72		98·9	145	159·1	44·19
62·14	91·14	100	27·78		99·4	145·8	160	44·43
64	93·87	103·0	28·61		100	146·7	160·9	44·70
64·77	95	104·2	28·96		102	149·6	164·2	45·60
65·24	95·7	105	29·17		102·3	150	164·6	45·72
66	96·8	106·2	29·50		102·5	150·4	165	45·83
68	99·7	109·4	30·40		104	152·5	167·4	46·49
68·18	100	109·7	30·48		105·6	154·9	170	47·22
68·34	100·2	110	30·54		105·7	155	170·1	47·24
70	102·7	112·7	31·29		106	155·5	170·6	47·39
71·46	104·8	115	31·94		108	158·4	173·8	48·28
71·59	105	115·2	32·00		108·7	159·5	175	48·61
72	105·6	115·9	32·19		109·1	160	175·6	48·77
74	108·5	119·1	33·08		110	161·3	177·0	49·17
74·55	109·3	120	33·32		111·8	164·0	180	50
75·0	110	120·7	33·53		112	164·3	180·2	50·07
76	111·5	122·3	33·97		112·5	165	181·1	50·29
77·67	113·9	125	34·72		114	167·2	183·5	50·96
78	114·4	125·5	34·87		115	168·6	185	51·39
78·41	115	126·2	35·05		115·9	170	186·5	51·81
80	117·3	128·7	35·76		116	170·1	186·7	51·86
80·77	118·4	130	36·10		118	173·1	189·9	52·75
81·82	120	131·7	36·57		118·1	173·2	190	52·78
82	120·3	132·0	36·66		119·3	175	192·1	53·34
83·89	123·0	135	37·50		120	176·0	193·1	53·64
84	123·2	135·2	37·55		121·2	177·7	195	54·17
85·23	125	137·2	38·10		122	178·9	196·3	54·54
86	126·1	138·4	38·44		122·7	180	197·5	54·86
86·98	127·6	140	38·87		124	181·9	199·5	55·43
88	129·1	141·6	39·34		124·3	182·3	200	55·56
88·64	130	142·6	39·62		125	183·3	201·2	55·88

In practical calculations it is rarely necessary to take into account the slight deviations from a constant value of the acceleration during the first few seconds after starting the train. This first section of the

speed-time diagram may consequently often be taken as practically a straight line. The straight-line portion is followed by a curved portion corresponding to a rapidly decreasing acceleration until, finally, a substantially constant speed is reached. This is shown in Fig. 5.

It is seen that in the example chosen, the substantially constant speed may be considered to have been attained at the end of 70 seconds, and is of the value of 35 ml ph. This maximum speed

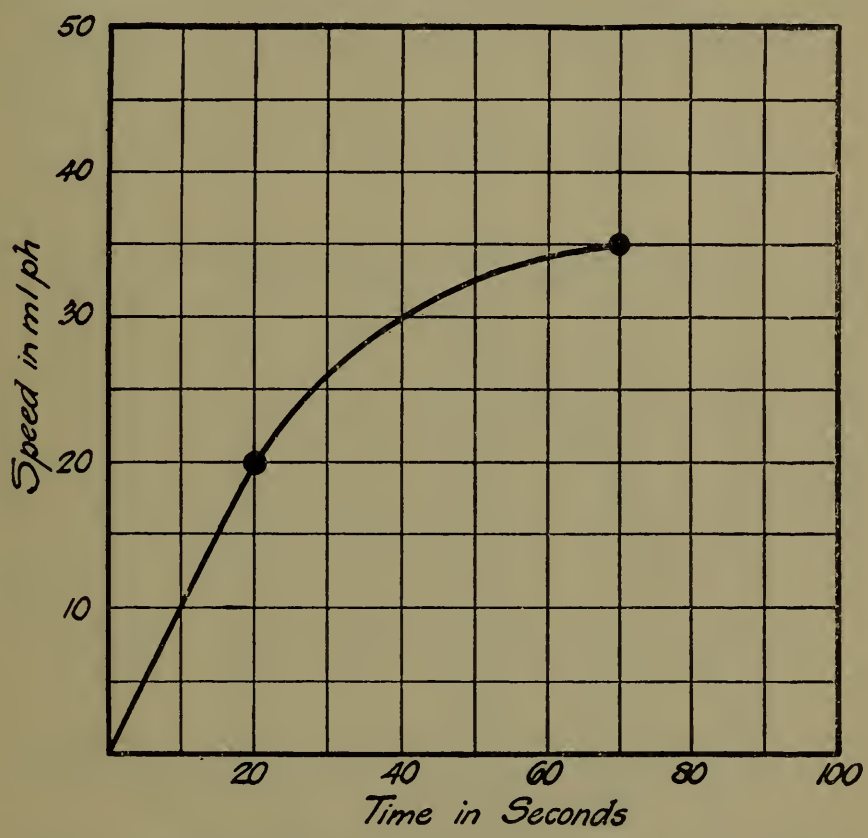

FIG. 5.—Portion of a Speed-time Diagram up to Point of *Crest Speed.*

attained during any run may be designated as the *crest* speed. The second section of the speed-time diagram, *i.e.* the section extending, in this case, from the 20th to the 70th second, is, for reasons which will be apparent later, termed the section corresponding to "running on the motor characteristic." In our example, the "straight-line" acceleration lasted for 20 seconds and was of the value of one ml phps. The "motor characteristic" acceleration lasted for 50 seconds and had the average value of—

$$\frac{35 - 20}{50} = 0{\cdot}30 \text{ ml phps.}$$

The shape of the portion of the curve corresponding to running on the "motor characteristic" is dependent on the design and type of the motor, and is not amenable to simple calculation.* If electricity continues to be supplied to the motors, the train will continue to accelerate slightly for a considerable time—say two to five minutes—and will not reach a constant speed until the power delivered to the wheels equals the power required to overcome the resistance to motion (*i.e.* the train-friction) at that speed. It is therefore seen that this constant speed running, only applies to

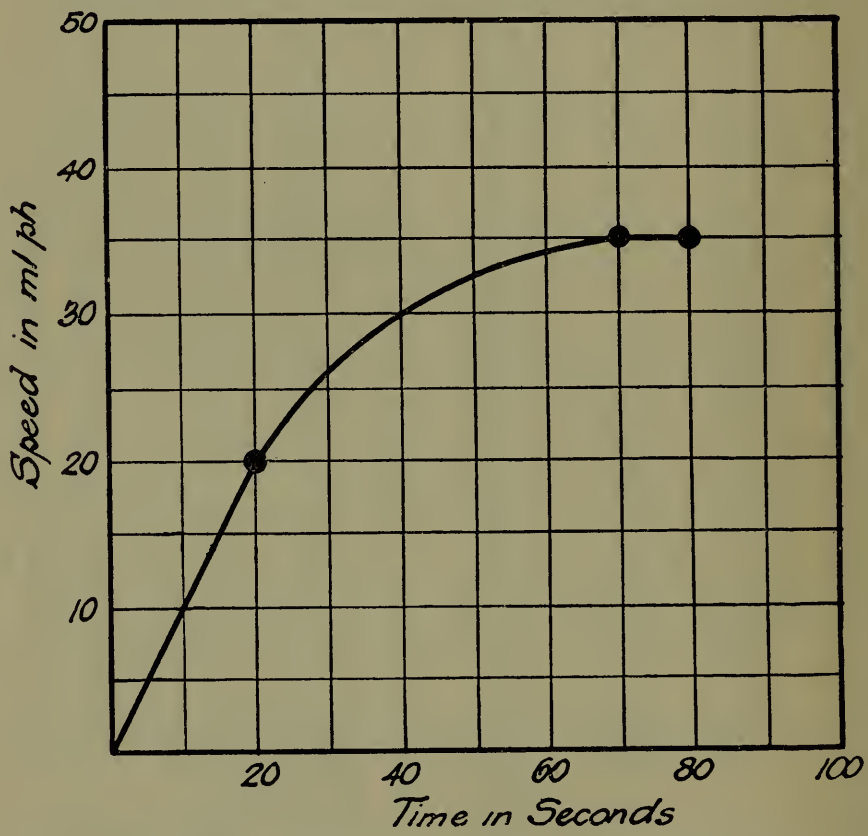

FIG. 6.—Portion of a Speed-time Diagram up to Point of "*Cut off*."

relatively long runs, but, in order to introduce into our speed-time diagram a part representative of this condition, we will assume that a substantially constant speed has been attained, and the train may be regarded as continuing to run at this speed for a short time, which, in the present instance, we may take as 10 seconds. Consequently, the shape of the speed-time diagram up to the 80th second from starting the train is that shown in Fig. 6. At this point, the electricity is *cut off* and the train is allowed to decrease in speed under the

* The "motor characteristic" is discussed more fully in Chapter X. p. 132.

retarding influence of friction. The train is said to "coast" or "drift." This "coasting" section of the speed-time diagram will usually be maintained until the train has nearly arrived at the point of the route where it is desired that it should stop. Let us assume, for our example, that the train coasts for 20 seconds. At the conclusion of the 20 seconds the speed will have fallen a small amount, the precise value of which is dependent upon the train-friction in any particular case. The fall in speed, with good rolling stock and track, is usually of the order of some 0·07 ml phps; *i.e.* the train-friction occasions a *deceleration* of 0·07 ml phps. Consequently, in our example, the speed at the end of the 100th second, *i.e.* at the end of the coasting section, will be—

$$(35 - 20 \times 0{\cdot}07 =)33{\cdot}6 \text{ ml ph},$$

as shown in Fig. 7.

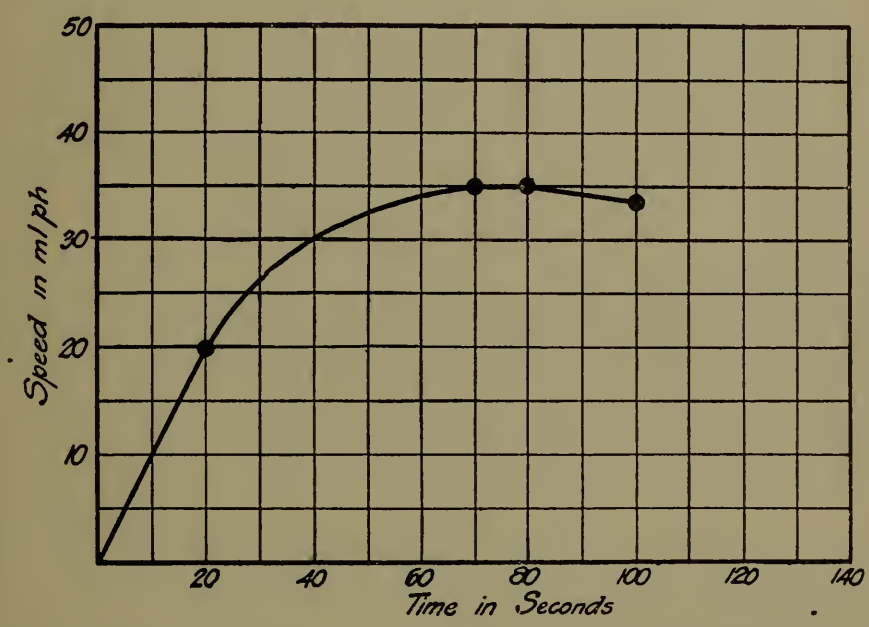

Fig. 7.—Portion of a Speed-time Diagram up to point of *Application of Brakes.*

The next and final section of the speed-time diagram is that during which the brakes are applied until the train is brought to rest. Let us, for example, consider that the brakes are applied with such a pressure as to occasion a deceleration of 1·5 ml phps. This is a deceleration which, if maintained uniform, does not involve discomfort to passengers. The duration of the "braking" section of the speed-time diagram is thus $\left(\frac{33{\cdot}6}{1{\cdot}5} =\right)22{\cdot}5$ seconds, as shown in Fig. 8. In normal services of city and suburban passenger trains, the brake

system is usually so operated as to occasion a deceleration of from 1 to 2 ml phps. The present tendency is toward still higher values of the deceleration.

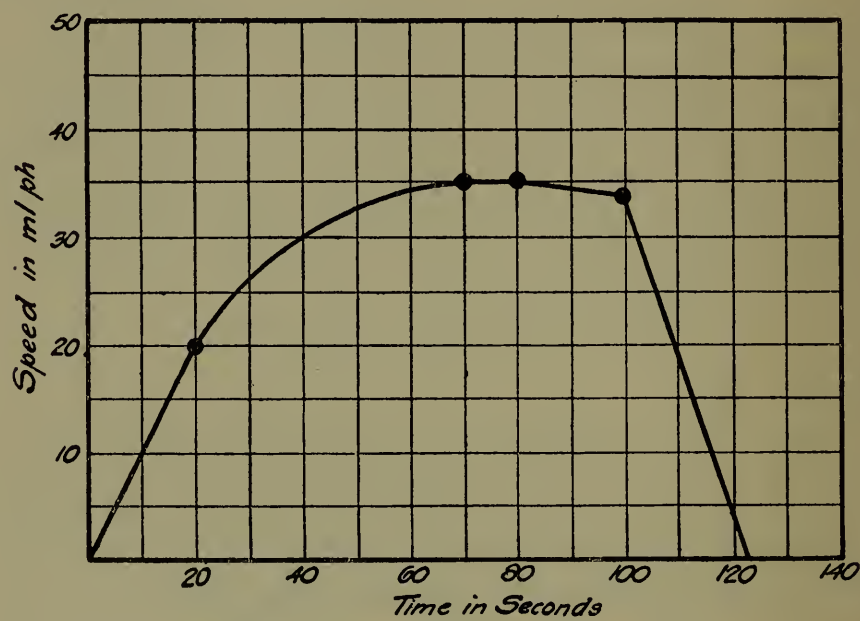

FIG. 8.—Complete Speed-time Diagram.

We now have the complete speed-time diagram. The run from start to stop has been accomplished in 122·5 seconds. The distance traversed may be obtained by the steps shown in Table III.

TABLE III.—DATA OBTAINED FROM THE SPEED-TIME DIAGRAM OF FIG. 8.

Section.	Duration (seconds).	Mean speed (ml ph).	Distance covered.	
			(mile).	(m).
"Straight-line" acceleration . .	20	10·0	0·0555	89·2
"Motor-characteristic" . . .	50	30·2	0·420	675·0
Constant speed	10	35·0	0·0975	156·5
Coasting (or drifting) . . .	20	34·3	0·190	305·5
Braking	22·5	16·8	0·1050	168·8
Total	122·5	—	0·868	1395

Thus, a total distance of 0·868 mile is covered in 122·5 seconds. The average speed is consequently—

$$\frac{3600 \times 0{\cdot}868}{122{\cdot}5} = 25{\cdot}5 \text{ ml ph.}$$

The mean speed from start to stop is termed the *average* speed. But the mean speed between two successive *starts*, i.e. the mean speed *including the time during which the train is at rest*, is termed the *schedule* speed. Thus, in the case of our example, if a 20-second stop is made at each station, then the time elapsing between successive starts (or stops) is $122{\cdot}5+20=142{\cdot}5$ seconds, and the *schedule* speed is—

$$\frac{3600 \times 0{\cdot}868}{142{\cdot}5} = 21{\cdot}9 \text{ ml ph.}$$

As characteristic of the particular speed-time diagram which we have taken for our example, we have—

Crest speed	35·0 ml ph.
Average speed	25·5 „
Schedule „	21·9 „

It is interesting to observe that in order to obtain a schedule speed of 21·9 ml ph it has been necessary to employ a crest speed of 35·0 ml ph. The crest speed is, in this case, 1·60 times the schedule speed. The crest speed is also 1·37 times the average speed.

The value of the ratio of the crest to the average speed is of great interest, and is a rough measure of the severity of a service. The higher the speed and the shorter the distance between stops, the greater must be the ratio of the crest to the average speed, and the more severe is the service. The ratio of the crest to the average speed does not depend exclusively on the average speed and the distance between stops, but also depends on the acceleration during starting and the deceleration during braking, the ratio being lower the higher the acceleration and deceleration. The subject of the ratio of the crest to the average speed is considered thoroughly in subsequent chapters.

In the speed-time diagram which has been employed as an example in the preceding pages (see Fig. 8), straight-line acceleration has been maintained up to 57 per cent. of the crest speed. This particular diagram has been convenient in serving the purposes of explaining the fundamental principles involved, but in practice straight-line acceleration often extends up to only some 50 per cent., or even less, of the crest speed. Moreover, for a service with frequent stops and high schedule speed, there is seldom an interval of running at constant speed. For such a service, certain rational assumptions are made as to the accelerating, coasting, and braking rates, and these are employed in constructing a representative speed-time diagram. In operating the train, it is usually economical to cut off the electricity immediately on the attainment of the crest speed given in the representative diagram. This speed will usually be well

below the maximum speed which would be attained by running longer on the "motor curve," but such relations depend on many conditions, which have to be taken into account when specifying the equipment for any particular service.

A speed-time diagram is shown in Fig. 9 which is typical for

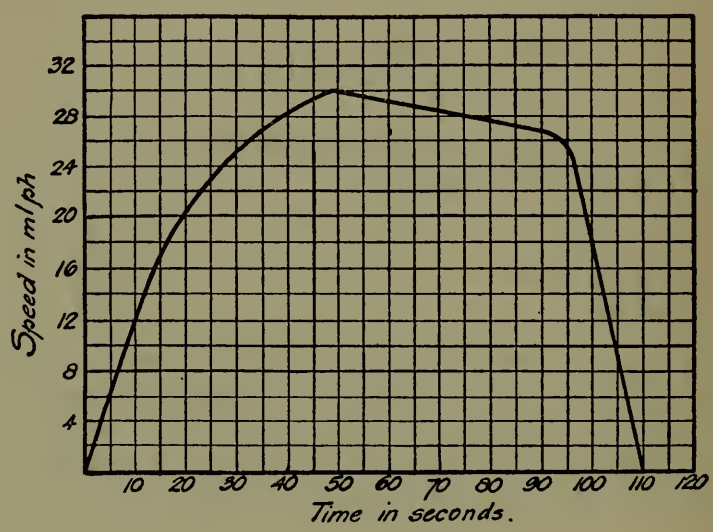

FIG. 9.—Speed-time Diagram for an Electric Train operating over a Distance o 1120 m.

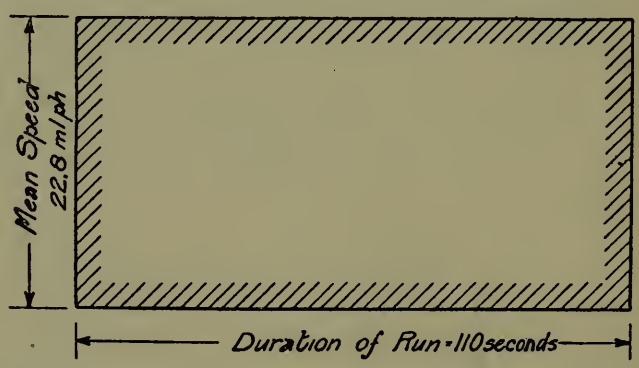

FIG. 10.—Area equal to that enclosed by the Diagram in Fig. 9, and therefore representing the Distance covered during the Run.

electric train operation under rather severe conditions, and which has no interval of running at constant speed, electricity being cut off 48 seconds from the start, and the train allowed to drift for the next 45 seconds, with consequent decrease in speed due to train resistance. The brakes are then applied, and the train is brought to rest 110 seconds from the instant of starting.

In Fig. 11 is given a speed-time diagram for a steam train operating over a route of the same number of stops per mile (that is, with the same distance between stops). The low acceleration shown in the diagram is typical of a steam-hauled train. The crest speed is the same in Fig. 11 as in Fig. 9, but, as a consequence of the lower

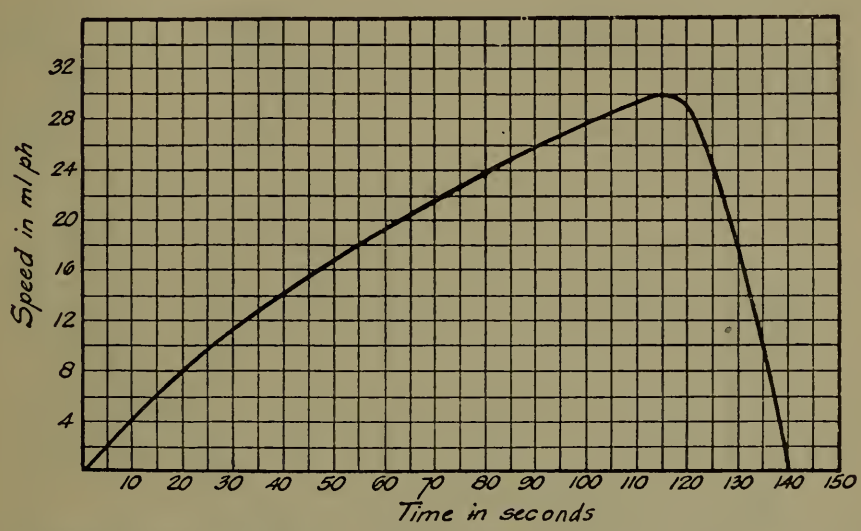

FIG. 11.—Speed-time Diagram for a Steam Train performing the same Journey as the Electric Train of Fig. 9.

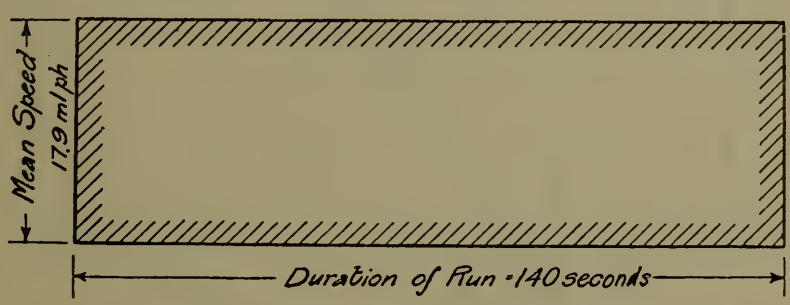

FIG. 12.—Area equal to that enclosed by the Diagram in Fig. 11, and therefore representing the Distance covered during the Run.

acceleration of the steam train, its speed from start to stop, *i.e.* its *average* speed, is much less than that of the electric train.

The distance from start to stop may be found from a speed-time diagram by measuring its area, either by means of a planimeter or else by obtaining the average ordinate for the entire diagram (which is, of course, equal to the average speed, in ml ph, from start to stop), and multiplying by the time, expressed with the hour as unit. Thus, from the diagrams of Figs. 9 and 11, the data set forth in Table IV. may be obtained—

TABLE IV.—DATA OBTAINED FROM FIGS. 9 AND 11.

	Electric Train (Fig. 9).	Steam Train (Fig. 11).
Average speed (ml ph) a	22·8	17·9
Time from start to stop (seconds) . .	110	140
,, ,, (hour) b	0·0306	0·0389
Distance from start to stop (mile), $a \times b$.	0·698	0·698
Do. (m) $a \times b \times 1609$	1120	1120
Crest speed (ml ph)	30	30
Ratio of crest to average speed . . .	1·32	1·68

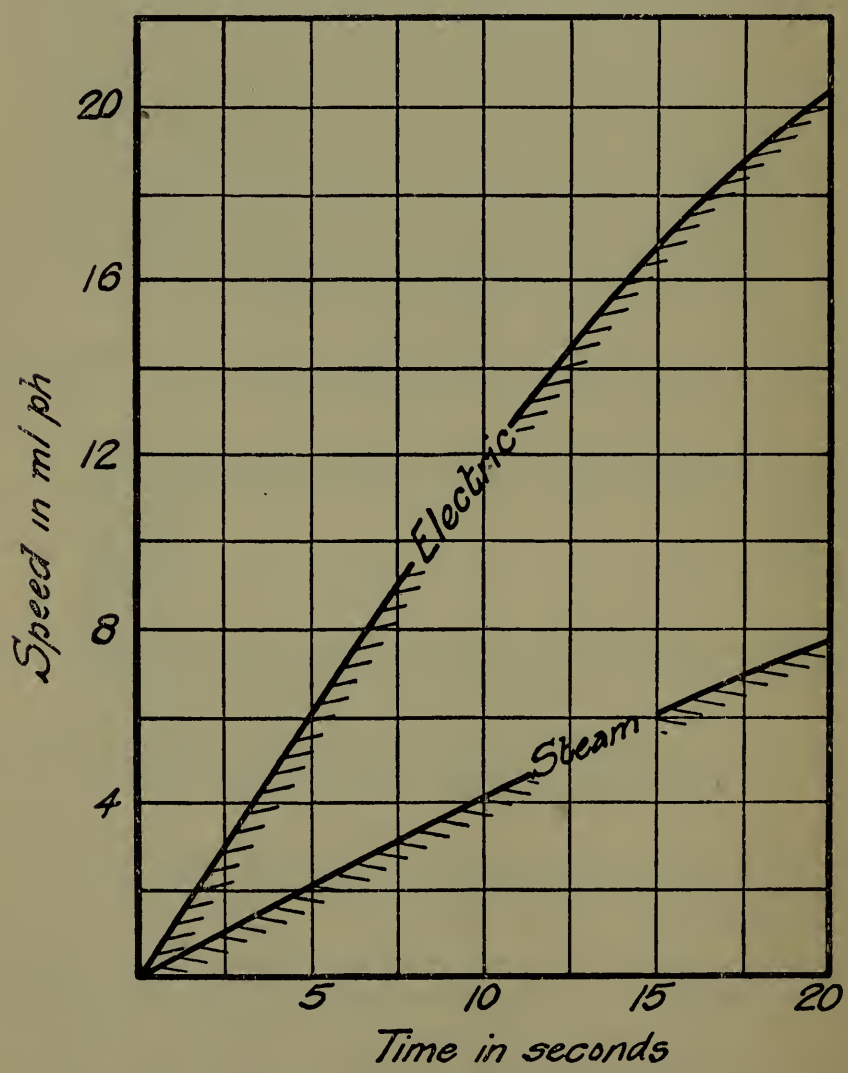

FIG. 13.—Initial Acceleration Periods of Speed-time Diagrams for the Electric and Steam Services of Figs. 9 and 11.

In Figs. 10 and 12 the two rectangles are respectively equal to the areas of the speed-time diagrams of Figs. 9 and 11. In these rectangles the ordinates equal the average speeds and the abscissae equal the times required to cover the distance from start to stop. Since the distance is the same in the two cases, the areas of these two rectangles are equal. Obviously, then, the time (represented by the length of the base of the rectangle) which is required to traverse a given distance is inversely proportional to the average speed (represented by the height of the rectangle) maintained during the journey.

The distance from the start which has been traversed up to any particular instant of the journey may be obtained by estimating

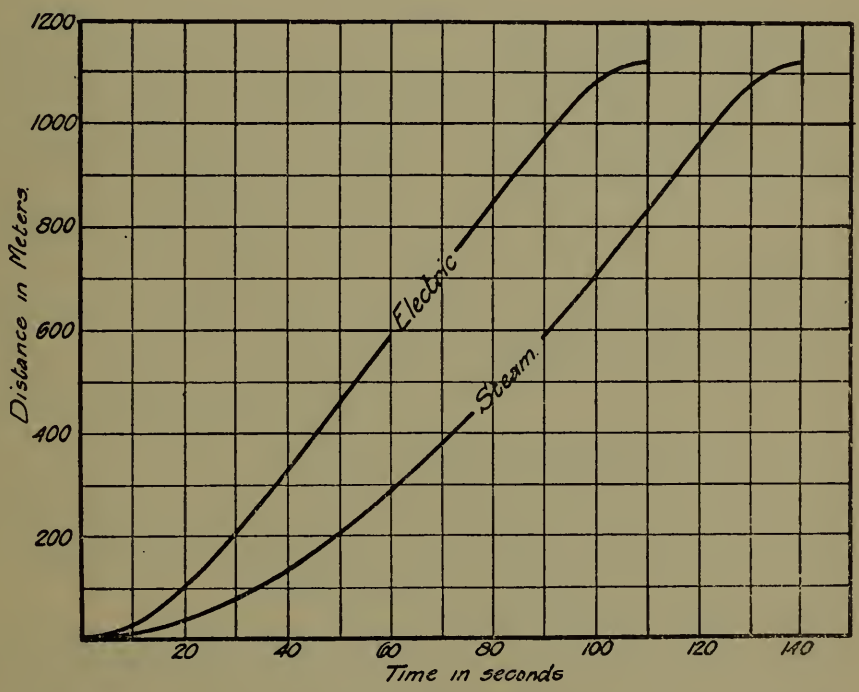

FIG. 14.—Distance-time Curves corresponding to Figs. 9 and 11.

the area of that portion of the diagram lying to the left of the ordinate corresponding to the instant in question. Thus, the distances covered during the first 20 seconds from the start in Figs. 9 and 11 are equal to the two areas shown in Fig. 13. The average ordinates of these two areas are respectively 11·2 and 4·0, *i.e.* the average speeds for the first 20 seconds from the start are respectively—

For Electricity	11·2 ml ph
„ Steam	4·0 ml ph.

The corresponding distances are—

$$\text{For Electricity, } 11{\cdot}2 \times \frac{20}{3600} \times 1609 = 100 \text{ m}$$

$$\text{„ Steam, } \quad 4{\cdot}0 \times \frac{20}{3600} \times 1609 = 36 \text{ m.}$$

In this way, curves may be derived in which distances (in m) are plotted as ordinates against time (in seconds) as abscissae. The distance-time curves corresponding to the speed-time diagrams of Figs. 9 and 11 are plotted in Fig. 14.

Examples.

1. If a train is accelerated for 8 seconds at the rate of 1·2 ml phps, what is its speed at the end of the 8th second? *Ans.* 9·6 ml ph.

2. What must be the average acceleration of a train for the first 20 seconds from the start if its speed at the end of that time is 18 ml ph? *Ans.* 0·9 ml phps.

3. What accelerations are usually employed on electric city and suburban railways operating with frequent stops? *Ans.* 0·8 to 1·5 ml phps.

4. Ditto when the trains are hauled by steam locomotives? *Ans.* 0·3 to 0·6 ml phps.

5. What circumstances limit in practice the acceleration of electric city and suburban trains? *Ans.* (As in text, p. 5).

6. What is the customary rate of braking of electric trains? *Ans.* 1·5 to 2·0 ml phps.

7. What deceleration is caused on a good level permanent way by the train friction when the supply of electricity is cut off and the train drifts? *Ans.* Some 0·07 to 0·09 ml phps.

8. If in a certain instance train friction occasions a deceleration of 0·07 ml phps, and if, when the train is running at a speed of 30 ml ph, the supply of electricity is cut off, what will be the speed of the train after it has "drifted" for 50 seconds? *Ans.* 26·5 ml ph.

9. For how many seconds would the train run, assuming the deceleration remained constant, if it were allowed to drift until it came to rest? *Ans.* 429 seconds.

10. What distance would it cover in this time? *Ans.* 1·79 mile (*i.e.* 2880 m).

11. When a train is running at a speed of 20 ml ph at the moment the brakes are applied, and it is desired to bring the train to rest in 15 seconds, what average deceleration must be occasioned by the brakes? *Ans.* 1·33 ml phps.

12. What distance will be covered, in this case, during the 15 seconds of application of the brakes? *Ans.* 0·042 mile (*i.e.* 67·6 m).

13. When the *schedule* speed of a train is 15 ml ph and there is 1 stop per mile, and if each stop is of 20 seconds' duration, what is the *average* speed of the train? *Ans.* 16·4 ml ph.

14. Plot a diagram, with distances as ordinates and times from start as abscissae, corresponding to the speed-time diagram of Fig. 8.

15. If, instead of as in Fig. 8, the deceleration from the 100th second until the train came to rest had been maintained at the constant value of 0·50 ml phps, (i.) what would have been the duration of the journey, from start to stop, in seconds? (ii.) What distance would have been covered? (iii.) What would have been the average speed? *Ans.* (i.) 167·3 seconds; (ii.) 1·077 mile (*i.e.* 1730 m); (iii.) 23·3 ml ph.

16. Plot the complete speed-time diagram and also a distance-time curve corresponding to the conditions of the last question.

CHAPTER II

THE INFLUENCE OF THE NUMBER OF STOPS PER MILE, AND OF THE DURATION OF EACH STOP

It is of the utmost importance, as affecting the earning capacity of a railway, that, for short and fast runs, the stops at stations shall be of the shortest practicable duration. To illustrate the importance of this point, let us consider the case of a train stopping every half mile, and operated with an average speed of 22 ml ph. If the stops were of 0 seconds duration, the schedule speed would be 22 ml ph.

The train stops every half mile. Consequently, the time occupied by a single run from start to *stop* is—

$$\frac{0{\cdot}5}{22} \times 3600 = 82 \text{ seconds.}$$

If the duration of each stop is 10 seconds, then the time from start to *start* is—

$$82 + 10 = 92 \text{ seconds,}$$

and the schedule speed will be only—

$$\frac{82}{92} \times 22 = 19{\cdot}6 \text{ ml ph.}$$

With 20-second stops the schedule speed will be reduced to—

$$\frac{82}{102} \times 22 = 17{\cdot}7 \text{ ml ph.}$$

Making similar calculations for stops of other durations, we arrive at the values set forth in Table V.—

Table V.—Influence of Duration of Stop on the Schedule Speed for an Average Speed of 22 ml ph, and for a Length of Run of 0·5 Mile.

Average speed (ml ph).	Length of run (mile).	Duration of stop (seconds).	Schedule speed (ml ph).	Percentage by which the schedule speed is less than the average speed (per cent.).
22	0·5	0	22·0	0
22	0·5	10	19·6	11·0
22	0·5	20	17·7	19·5
22	0·5	30	16·1	26·8
22	0·5	40	14·8	32·7
22	0·5	50	13·7	37·7
22	0·5	60	12·7	42·3

With less frequent stops, *i.e.* with longer runs, the influence for the same average speed is much less pronounced. Thus, for 1 stop per mile, and an average speed of 22 ml ph, the results are shown in Table VI.—

TABLE VI.—INFLUENCE OF DURATION OF STOP ON THE SCHEDULE SPEED FOR AN AVERAGE SPEED OF 22 ML PH, AND FOR A LENGTH OF RUN OF 1 MILE.

Average speed (ml ph).	Length of run (mile).	Duration of stop (seconds).	Schedule speed (ml ph).	Percentage by which the schedule speed is less than the average speed (per cent.).
22	1·0	0	22·0	0
22	1·0	10	20·8	5·5
22	1·0	20	19·6	11·0
22	1·0	30	18·6	15·5
22	1·0	40	17·7	19·5
22	1·0	50	16·9	23·2
22	1·0	60	16·1	26·8

The effect is also less serious the lower the average speed. Thus, with 2 stops per mile, and an average speed of only 11 ml ph, the results are as shown in Table VII.—

TABLE VII.—INFLUENCE OF DURATION OF STOP ON THE SCHEDULE SPEED FOR AN AVERAGE SPEED OF 11 ML PH, AND FOR A LENGTH OF RUN OF 0·5 MILE.

Average speed (ml ph).	Length of run (mile).	Duration of stop (seconds).	Schedule speed (ml ph).	Percentage by which the schedule speed is less than the average speed (per cent.).
11	0·5	0	11·0	0
11	0·5	10	10·4	5·5
11	0·5	20	9·80	11·0
11	0·5	30	9·30	15·5
11	0·5	40	8·85	19·5
11	0·5	50	8·45	23·2
11	0·5	60	8·05	26·8

In Fig. 15 are plotted, for various durations of stop, curves with schedule speeds as ordinates and average speeds as abscissae, for a 1-mile run from start to stop. From these curves we see that while, with 10-second stops, a schedule speed of 25·0 ml ph requires an average speed of 27·0 ml ph, the corresponding average speed when the stops are of 30 seconds' duration is 31·5 ml ph. In Fig. 16 ordinates and abscissae still represent respectively schedule and average speeds, but each curve relates to some stated distance between stations. The

full-line curves all relate to 10-second stops, and the dotted line curves all relate to 30-second stops.

It has been found by experience that for such cases as the London underground railways the average duration of stop need not exceed

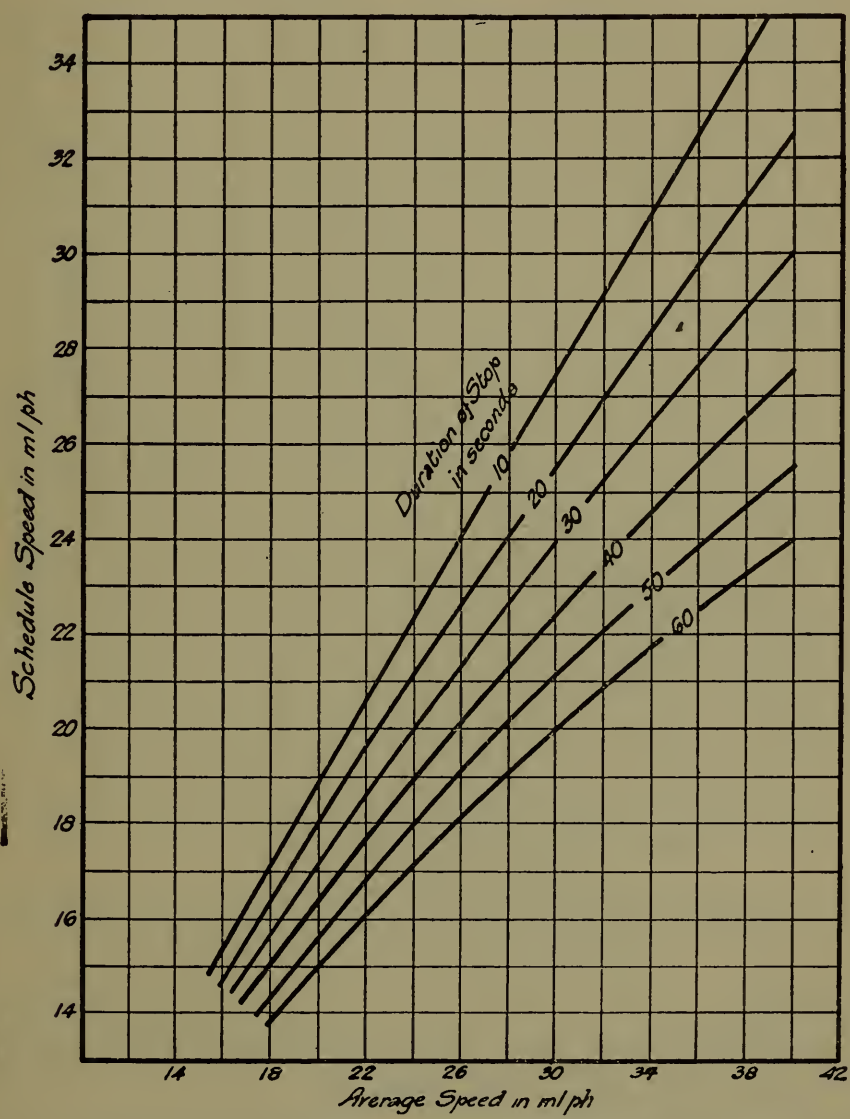

FIG. 15.—Relation of Schedule to Average Speed for Various Durations of Stop for a 1-mile Run from Start to Stop.

20 seconds; at certain stations and times the stops must be longer, at others they may be shorter.

The values in Tables V., VI., and VII., and the curves in Figs. 15 and 16, have been derived by starting from a given average speed and calculating the schedule speed. The reverse process, where the required *schedule* speed is given, and it is desired to ascertain the corresponding

average speed, may be carried out by reading off the values from curves such as those of Figs. 15 and 16, or by calculations of the following type.

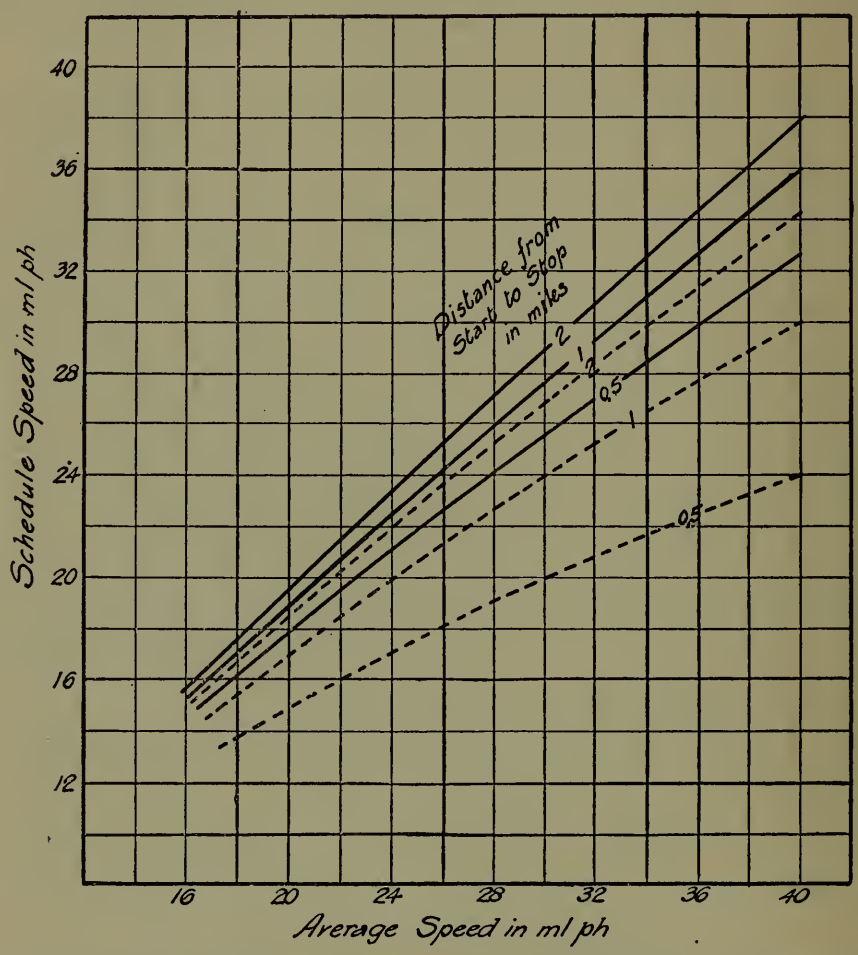

FIG. 16.—Relation of Schedule to Average Speed for Various Lengths of Run with 10-second and 30-second Stops.

10 seconds . . . . . Full line curves.
30 seconds . . . . . Dotted line curves.

Take the case of a *schedule* speed of 15 ml ph and one stop per mile. The time from *start* to *start* will be—

$$\frac{3600}{15 \times 1} = 240 \text{ seconds.}$$

With a 10-second stop, the time from *start* to *stop* must be—

$$240 - 10 = 230 \text{ seconds.}$$

Consequently, the average speed is—

$$\frac{240}{230} \times 15 = 15{\cdot}7 \text{ ml ph.}$$

For a given schedule speed, the number of stops per mile constitutes a very important factor as bearing upon the *severity* of the service, owing to the extent to which it affects the ratio of the crest to the average speed, and also (as we shall see in following chapters) the energy consumption per ton-mile.

It is profitable at this point to anticipate for a moment and draw attention to the two curves in Fig. 17. These curves relate respectively to runs of 0·5 mile and 1 mile between stations, and show the dependence of the highest schedule speed attainable in practice on the duration of each stop. The curves are based on 1·5 ml phps as the mean of the acceleration and deceleration, and the *crest* speed is

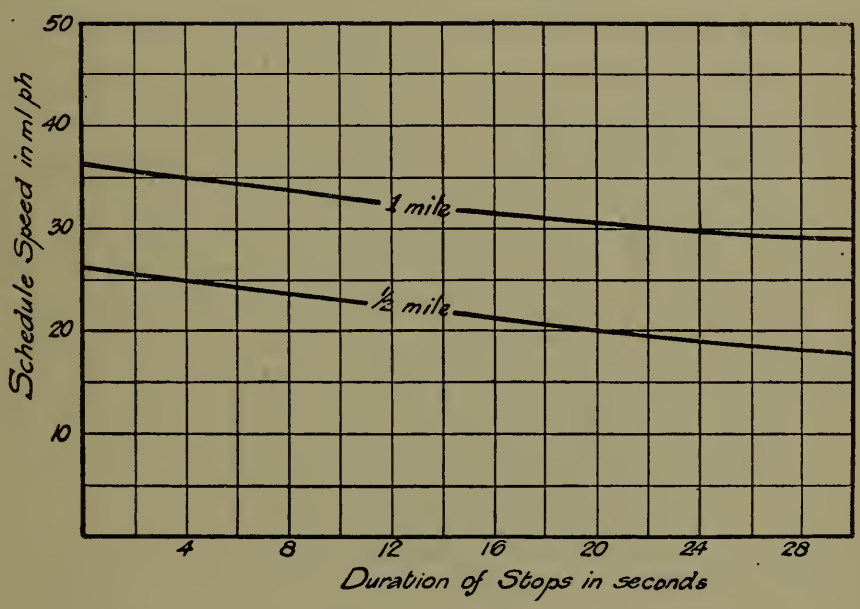

FIG. 17.—Curves showing Influence of Duration of Stop on the Schedule Speed for Runs of 0·5 and 1-mile Distance from Start to Stop.

taken as about 1·7 times the *average* speed. These, while attainable, are rather extreme conditions, and commercial limitations would usually lead to decreasing by from 10 per cent. to 20 per cent. the values to which the curves are plotted. The curves serve, however, to show very clearly the importance of operating trains for a city and suburban service with as brief stops as possible at stations.

It is usually assumed that "moving platform" schemes of passenger transportation are impracticable. While grave difficulties are, of course, inherent to such schemes, there are, nevertheless, strong grounds for giving them careful consideration. Thus, from Fig. 17 we see that for 0·5-mile runs between stops, and with 20-second stops, the highest practicable *schedule* speed is some 20 ml ph. As a

matter of fact, the *commercial* limit is more like 17 ml ph. A train operating to a schedule speed of 17 ml ph makes—

$$\frac{17}{0{\cdot}5} = 34 \text{ stops per hour.}$$

Consequently, the train is at rest for (34 × 20 =)680 seconds out of the 3600 seconds in one hour, and its *average* speed is—

$$\frac{3600}{3600 - 680} \times 17 = 21{\cdot}0 \text{ ml ph.}$$

Its *crest* speed will be some (1·6 × 21·0 =)33·6 ml ph. If the train weighs 200 tons, then the engineering proposition involves imparting to a weight of 200 tons a speed of 33·6 ml ph, and bringing this weight to rest again every $\left(\frac{3600}{34} = \right)106$ seconds, and the result consists in being able to transport passengers at 17 ml ph. The moving-platform alternative for transporting passengers at this same speed, only involves a *maximum speed of any moving part* of 17 ml ph. Even if there were required three other platforms, moving respectively at 15, 10, and 5 ml ph, these platforms need not extend over the entire route. Indeed, they may be confined to stations, and may be concentric platforms moving over an elliptical route, and surrounding an island platform to which the passengers gain access by lifts or stairs. A system of this sort might ultimately lead to such sound engineering solution (as the result of the study of successive constructions) as to ultimately result in running trains at a constant speed—say, 17 ml ph—and arranging for the passengers to embark and disembark from travelling platforms at the stations. The only energy required in such a system is that employed in overcoming friction. The enormous amount of energy required in a high-speed service with frequent stops, for imparting momentum to the train, many times per hour, would be saved. Of course, instead of platforms at the station, moving over an elliptical route, other arrangements might be adopted, such as platforms in forms resembling endless belts, those portions which, for the time being, constitute the upper side being accessible to the passengers. While various types of moving platforms are already familiar accessories of modern life, nevertheless, these suggestions are not made with the serious thought of their early realization, but simply for the purpose of preparing the reader to better appreciate that the bulk of the energy consumed in train-propulsion is, in high-speed, frequent-stop services, required to supply train momentum, and with present methods, a very large part of the energy of momentum is ultimately wasted at the brake-shoes, as heat. The subject is given thorough consideration in subsequent chapters. For the present it may be stated that whereas a train travelling at

a *constant* speed of 17 ml ph need not consume, at the outside, more than some 18 w hr per ton-mile, the maintenance of a schedule speed of 17 ml ph, with 2 stops per mile, would involve a consumption of at least 90 w hr per ton-mile, *i.e.* a consumption at least 5 times as great per ton-mile as the constant-speed proposition.

If the total weight of the ordinary (17 ml ph) stopping-train is 200 tons, some 15 per cent. of this weight—or, say 30 tons—represents the weight of the electrical equipment, and 170 tons represents the weight of the trucks, under-frames, and coach bodies. Owing to the far lower stresses in a constant-speed train, the weight of the trucks, under-frames, and coach bodies would, for the same seating capacity, be not more than, say 120 tons, and the weight of the electrical equipment would come down to less than 5 tons, making the total weight of the constant-speed train not over 125 tons. The cost of the two trains would be some £16,000 and £10,000 respectively. The £6000 saved in the capital cost per train will go far to defray the additional capital outlays at stations. The train consumptions will, in the two cases, be respectively $\frac{90 \times 200}{1000} = 18{\cdot}0$ kelvins * per train-mile for the stopping-train, and $\frac{18 \times 125}{1000} = 2{\cdot}25$ kelvins per train-mile for the constant-speed train.

At a price at the train, of 0·8*d.* per kelvin, the outlay for electricity for propelling the trains will be respectively—

14·4*d.* per train-mile for the stopping-train, and
1·8*d.* „ „ „ constant-speed train.

The electricity consumed in operating the apparatus at the stations would hardly be likely to exceed 3·0*d.* per train-mile, thus leaving a saving on the outlay for electricity of—

$$14{\cdot}4 - 3{\cdot}0 - 1{\cdot}8 = 9{\cdot}6d. \text{ per train-mile.}$$

I wish again to state emphatically that I am not to be understood as putting forward such propositions as commercial. There are many drawbacks, amongst which the engineering difficulties, while not insuperable, are certainly great. The savings in one direction might easily, as the result of a really thorough comparison, be much more than off-set by the increased outlays and operating costs in other directions. I have drawn attention to the future possibilities of such projects, with the present purpose of preparing the reader's mind to grasp certain important conclusions which will be dealt with in subsequent chapters. Two amongst these conclusions are—

I. The preponderating influence of the momentum in problems relating to high-speed trains making frequent stops.

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* One kelvin equals one kw hr, *i.e.* 1000 w hr.

II. Consequent upon Conclusion I. follows the second conclusion, namely, that for high-speed trains making frequent stops, it is of great commercial advantage to employ such systems of propulsion as shall, other things being equal, consist with low total weight of train for a given seating capacity.

Examples.

1. On a railway with an average distance of one mile between successive stations, the schedule speed is 24 ml ph, and the duration of stop is 10 seconds. From Fig. 15, find what duration of stop can be permitted in order to obtain the same schedule speed of 24 ml ph when the average speed is increased to 30 ml ph. *Ans.* 30 seconds.

2. An average speed of 20 ml ph and one 20-second stop per mile correspond to a schedule speed of 18 ml ph. Find the effect on the schedule speed of doubling the average speed and duration of stop.

Ans. Schedule speed, 27·5 ml ph; Ratio of average to schedule speed, 1·45.

3. Passing from a suburban line with stations every 0·5 mile, a train enters a section with stops every two miles. Find the possible reduction in the average speed if the duration of stop is constant at 30 seconds, and the schedule speed constant at 20 ml ph. (Use Fig. 16.)

Ans. 30 ml ph to 21·8 ml ph; 27 per cent. reduction.

4. A train is timed to traverse a 2-mile section in 5 minutes, but it has to make two stops at signals, of 10 seconds' and 30 seconds' duration respectively. What is the actual average speed? Compare with the average speed for the normal run.

Ans. 27·2 ml ph; normal, 24 ml ph; 13 per cent. increase.

5. Find from Fig. 15 the average speeds required on a 1-mile run to give a schedule speed of 26 ml ph when the stops are 15, 25 and 35 seconds respectively. *Ans.* 29·5, 32·0, 35·0 ml ph.

6. The distance between two stations is 1000 m, and has to be covered in 2 minutes, which includes a 30-second stop. Find the schedule and average speeds. *Ans.* 18·6; 24·9 ml ph.

7. For 0·5, 1 and 2-mile runs determine the ratios of schedule to average speed for stops of 10 seconds and 30 seconds, at a constant schedule speed of 24 ml ph.

Ans. (10 seconds) 0·975; 0·930; 0·864. (30 seconds) 0·910; 0·80; 0·60.

8. Repeat question 7 for other schedule speeds and plot a series of curves for 10 and 30-second stops, and 0·5, 1 and 2-mile runs, taking schedule speeds as abscissae and the ratios of schedule to average speed as ordinates. There will be 6 curves in all.

9. From these curves ascertain the ratio of schedule to average speed for a 1-mile run with a 30-second stop, and a schedule speed of 21 ml ph.

Ans. 0·82.

CHAPTER III

THE PREPONDERATING INFLUENCE OF MOMENTUM IN A SERVICE WITH FREQUENT STOPS

For long runs with but few stops, the energy required to propel the train is mainly devoted to overcoming friction. But this is not the kind of service for which electrical operation offers much, if any, advantage over steam operation, except in special cases, such, for instance, as to do away with smoke in tunnels, and to traverse routes with heavy grades. The most appropriate field for railway electrification is at present for services of very frequent trains, operating at fairly high speed and with frequent stops. For this class of service, only a small portion of the energy employed in propelling the train is required to overcome friction. Most of the energy is employed to impart momentum to the train during the acceleration period, and is subsequently transformed into heat during the coasting and braking periods, *i.e.* during those periods when the supply of electricity has been "cut off." Thus we shall find that certain very important aspects of the problems involved are best understood by first assuming the imaginary case where the train has no friction other than the friction between the brake-shoes and the wheels. Let us further assume that the controller arrangements are such that we have "straight-line" acceleration right up to the crest speed. With these two assumptions, our speed-time diagram consists of only three sections, as shown, for a hypothetical case, in Fig. 18.

In this hypothetical case, the first section occupies 24 seconds, and is devoted to "straight-line" acceleration at the rate of 1 ml phps. Thus, at the end of 24 seconds the speed is 24 ml ph. The electricity is then cut off, and the train drifts. Since it is assumed that there is no friction prior to the application of the brakes, the train continues running at constant speed. This "drifting" section is continued for 25 seconds. At the end of the $(24 + 25 =)$49th second from the start the brakes are applied at such a pressure as to decelerate the train at the rate of 1·5 ml phps. Consequently, since the train's speed is reduced every second by 1·5 ml ph the train will be brought to rest $\left(\frac{24}{1{\cdot}5} =\right)$16 seconds

after the instant of application of the brakes. Thus, at the end of the (24 + 25 + 16 =)65th second from the start, the train is brought

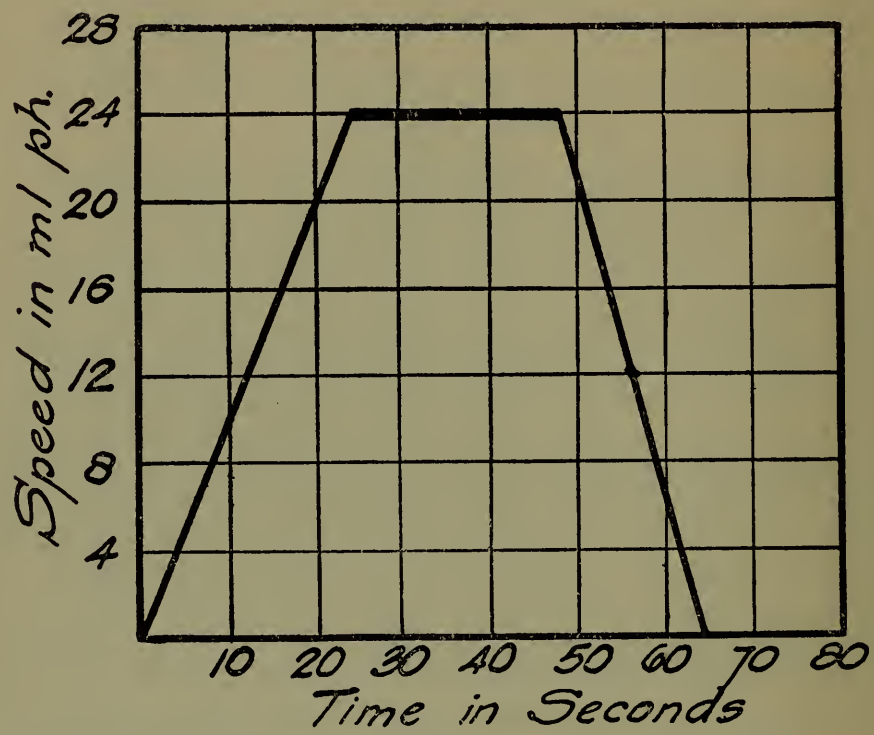


Fig. 18.—Hypothetical Speed-time Diagram assuming no Track Friction.

to rest. The distance traversed may be obtained as shown in Table VIII.—

Table VIII.—Data obtained from the Speed-time Diagram of Fig. 18.

Section.	Duration (seconds).	Mean speed (ml ph).	Distance covered (miles).
"Straight-line" acceleration .	24	12	0·080
Drifting	25	24	0·166
Braking	16	12	0·054
Total	65	—	0·30

A total distance of 0·30 mile is thus traversed in 65 seconds. The *average* speed for the entire run from start to stop is consequently—

$$\frac{3600}{65} \times 0{\cdot}30 = 16{\cdot}6 \text{ ml ph.}$$

Taking the duration of each stop as 20 seconds, we find that the *schedule* speed is—

$$\frac{65}{65 + 20} \times 16{\cdot}6 = 12{\cdot}7 \text{ ml ph.}$$

During the first section of each run (*i.e.* the section devoted to "straight-line" acceleration), a speed of 24 ml ph is imparted to the train. This is the period during which the electric motors on the train absorb energy from the line. Let us investigate the amount of energy which is stored up in the train during this period.

In imparting to one ton a given speed S (expressed in ml ph), E, the energy of translational momentum expressed in w hr, which is absorbed by the ton, may be obtained from the formula—

$$E = 0{\cdot}0278\, S^2 \,*$$

In our case, where the crest speed is 24 ml ph, the energy stored up as momentum per ton weight of train, in virtue of the *translational* motion, is consequently—

$$(0{\cdot}0278 \times 24^2 =)16{\cdot}1 \text{ w hr.}$$

In virtue of the momentum of the rotating parts, such, for instance, as the wheels and the armatures of the motors, *i.e.* in virtue of the energy of *rotational* motion, the total energy of momentum per ton weight of train will be some 9 per cent. greater than the above value.†

---

* The formula may be derived as follows :—

$$\text{Energy of momentum in kg m} = \tfrac{1}{2} \times \frac{\text{weight in kg}}{9{\cdot}81} \times (\text{speed in m ps})^2.$$

∴ For a weight of one ton (1000 kg), we have—

$$\text{Energy of momentum in kg m} = \frac{1000}{2 \times 9{\cdot}81} \times (\text{speed in m ps})^2.$$

But one w hr = 367 kg m
And one ml ph = 0·447 m ps

$$\therefore 367\, E = \frac{1000}{2 \times 9{\cdot}81} \times 0{\cdot}447^2 \times S^2$$

$$\therefore E = \frac{1000 \times 0{\cdot}200}{2 \times 9{\cdot}81 \times 367} \times S^2$$

$$\therefore E = 0{\cdot}0278\, S^2. \qquad \text{Q.E.D.}$$

† The energy due to the rotating parts will vary according to the design of the motors, wheels, etc. It will generally be slightly higher for trains operated by single-phase motors than for trains operated by continuous motors. Wilson and Lydall, in "Electrical Traction," vol. i. p. 374, state that the energy due to the rotating parts will generally vary between 6 to 10 per cent. of the total weight of train. The subject is discussed at length in a paper entitled "A Consideration of the Inertia of the Rotating Parts of a Train," by N. W. Storer ("Transactions of the American Institute of Electrical Engineers," vol. xix. p. 165). Carter, in his contribution to a discussion on "Electric Railways," at the Institution of Civil Engineers ("Proceedings, Institution of Civil Engineers," vol. clxxix. pt. 1), gives the following method

Thus, the total energy of momentum of the train, when it is running at a speed of 24 ml ph, is—

$$(1{\cdot}09 \times 16{\cdot}1 =)17{\cdot}5 \text{ w hr per ton,}$$

or

$$17{\cdot}5 \times 3600 = 63{,}000 \text{ w sec per ton.}$$

Since this amount of energy is absorbed by the train during the 24 seconds occupied in accelerating the train, the *rate* at which the energy is being stored up as momentum by the train, averaged over the entire accelerating period, is—

$$\frac{63{,}000 \text{ w sec}}{24 \text{ seconds}} = 2620 \text{ watts per ton; or } 2{\cdot}62 \text{ kw per ton.}$$

But the *average* rate at which energy is being stored up in the train as momentum, *when spread over the* 65 *seconds during which the train is in motion*, is—

$$\frac{63{,}000 \text{ w sec}}{65 \text{ seconds}} = 970 \text{ watts per ton; or } 0{\cdot}97 \text{ kw per ton.}$$

Including a 20-second stop at each station, the average rate at which energy is being stored up in the train as momentum, *taken over the whole route*, is—

$$\frac{65}{65 + 20} \times 970 = 740 \text{ watts per ton; or } 0{\cdot}74 \text{ kw per ton.}$$

Thus we have—

Energy absorbed, per ton, in momentum—

(*a*) Averaged over the time during which the train is taking electricity from the line . . . 2·62 kw.

(*b*) Averaged over the time during which the train is in motion . . . . . 0·97 kw.

(*c*) Averaged over the entire route . . . . 0·74 kw.

If the train in question has a total weight of 100 tons, the average rate at which energy is stored up in the train as momentum, *taken over the whole route*, is—

$$100 \times 0{\cdot}74 = 74 \text{ kw.}$$

---

of allowing for rotational momentum when there is no special knowledge of the moments of inertia of the respective train parts.

Add to the weight of the train an amount equal to—

$$\left[(0{\cdot}6 \times w) + 0{\cdot}5 \times a \times \left(\frac{d}{\mathrm{D}} \times g\right)^2\right]$$

where w = weight of all train wheels,
a = weight of all armatures,
d = outer diameter of armature,
D = diameter of driving wheel,
g = gear ratio.

The total momentum energy (including rotational) can then be estimated simply as energy for linear momentum on the above estimated total weight.

In the case of a road operating 100 such trains simultaneously, from a single power-house, the peaks of load will be so distributed that the rate at which energy is being stored up as momentum in these 100 trains will be fairly uniform at—

$$100 \times 74 = 7400 \text{ kw.}$$

Modification of the Results as regards Consumption per Train when Friction is taken into Consideration

When friction is considered, the drifting portion of the speed-time diagram will no longer be a horizontal line representing constant speed, but the line will slope so as to represent a drift-deceleration of 0·07 ml phps.

This change has been made in Fig. 19, where it is seen that the

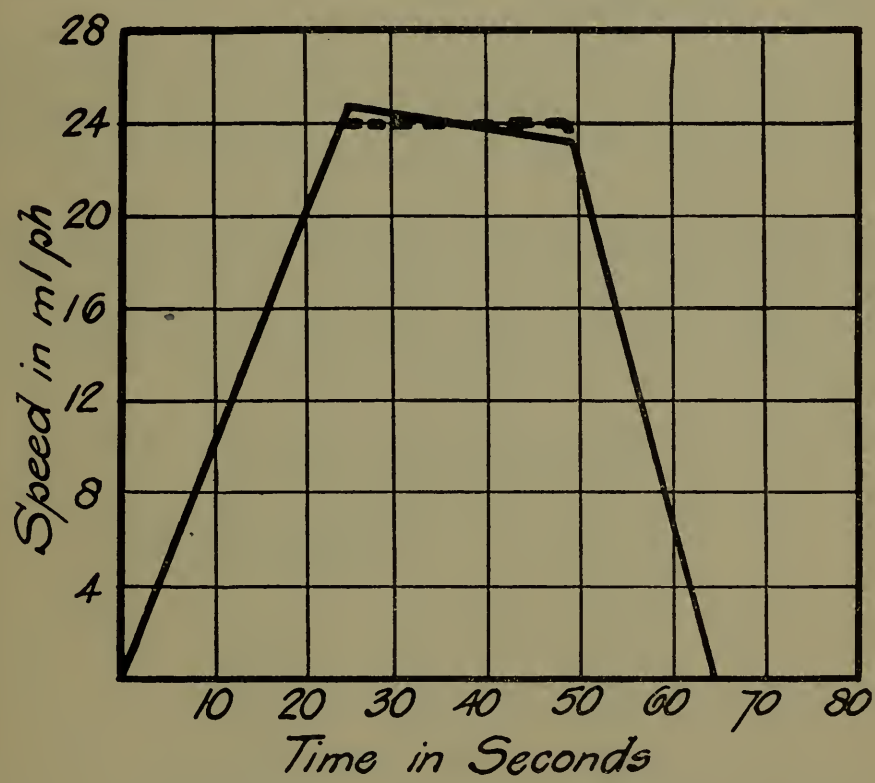


Fig. 19.—Hypothetical Speed-time Diagram taking Track Friction into Account.

distance covered can be maintained at the same value by raising the crest speed, and lowering the speed at which braking commences, thus keeping the same area enclosed by the speed-time diagram.*

* It has already been explained, on p. 15 of Chapter I., that the distance traversed is equal to the area of the speed-time diagram.

These changes would, however, make only slight differences in the energy considerations which follow, and it is not worth while taking them into account, since the conditions of this schedule are purely hypothetical, and are intended only to assist in describing useful ideas and methods of carrying out train-movement calculations.

The energy stored up in the train as momentum is subsequently transformed by friction into heat. This friction occurs at the bearings of the rotating parts, at the points of contact of wheel and track, at the ends and sides of the train, and, chiefly, at the brake-shoes.

It was asserted at the commencement of this chapter that, when trains are operated at high schedule speed and with frequent stops, much the greater part of the energy consumed at the train is required for providing the momentum corresponding to the maximum speed of the train, and that the energy required for overcoming train-friction is, in comparison, only a small amount. Let us now examine the correctness of this assertion.

The quantity of energy transformed into heat at the bearings of the rotating parts, and at the points of contact of wheel and track, may be considered to be inevitably associated with the propulsion of the train, and the quantity of energy consequently is, in a sense, effectively employed. The energy transformed into heat at the brake-shoes may, on the other hand, be regarded as wasted.

If the rolling stock is of good design, then, when the train is operated *at constant speed*, on a straight, level, and well-built permanent way, the tractive force required per ton-weight of train may, for a train of 100 tons weight, be taken at the values set forth in Table IX.—

TABLE IX.—TRACTIVE FORCE REQUIRED AT AXLES TO OVERCOME TRAIN RESISTANCE AT VARIOUS SPEEDS.

Speed (ml ph).	Tractive force required to overcome train resistance (kg per ton).
10	1·5
20	2·5
30	3·6
40	4·7
50	6·6
60	8·3
80	12·8
100	18·5

There are 1609 m in one mile.* Furthermore, 367 kg m equal

* One mile = 1609·3 m. The figure may be taken as 1609 or 1610, according to the degree of accuracy required, *i.e.* according as the calculation justifies 4 or only 3 significant figures.

1 w hr. From these two constants, and the data in the preceding table, we may compile the results in Table X.—

TABLE X.—ENERGY REQUIRED AT AXLES TO OVERCOME TRAIN FRICTION AT VARIOUS SPEEDS.

Speed (ml ph).	Energy required at axles to overcome train-friction (kg m per ton-mile).	Energy required at axles to overcome train-friction (w hr per ton mile).
10	2,410	7
20	3,700	10
30	5,800	16
40	7,550	21
50	10,600	29
60	13,300	36
80	20,600	56
100	29,800	81

These are rough estimates of the amounts of energy which would be required at the axles to overcome friction in propelling well-built trains *at constant speed* over a well-built, straight, and level track.

But when the speed of a train is rapidly changing throughout the journey, as in the case of a service with frequent stops, these train-friction data are found not to apply in practice. The train-friction is considerably higher than would be inferred from these data when taken to correspond to the mean speed of the train. We shall introduce no serious error into the results obtained in estimating the total energy consumed at the train, if we take the train-friction at a liberally high value. In fact, we are justified in simplifying the calculations by taking the tractive force per ton as of a constant value throughout the journey. A convenient and safe value to assume for the tractive force, for trains operating with frequent stops and with a *crest* speed of not over some 45 ml ph, is 6 kg per ton, even though the mean or *average* speed may be only some 15 to 30 ml ph. We thus see that the tractive force is taken at fully twice the value shown in the table, for a constant speed equal to the mean speed. In our example, namely, that of a 100-ton train running to a schedule speed of 12·7 ml ph, and with a stop every 0·30 mile, we may calculate the energy required at the axles as follows :—

Tractive force = 6 kg per ton.

Train-friction $= 6 \times 1609 = 9650$ kg m per ton-mile

$$= \frac{9650}{367} = 26{\cdot}2 \text{ w hr per ton-mile.}$$

We have seen in Table VIII. that the distance covered during the time that the train is taking electricity from the line, *i.e.* during the 24 seconds occupied by acceleration, is 0·080 mile. Consequently, the electricity required to overcome the train-friction during the first 24 seconds is—

$$26{\cdot}2 \times 0{\cdot}080 = 2{\cdot}10 \text{ w hr per ton.}$$

During the remaining 41 seconds of the run from start to stop, the electricity is cut off from the train, and the momentum of the train is drawn upon as the source of power to overcome train-friction during the remainder of the journey. During the 20-second stop at the station, there is, of course, no train-friction. Thus, every $(24 + 41 + 20 =)85$ seconds, 2·10 w hr are taken from the line by the train, to overcome train-friction, and in the course of one hour the total amount of electricity consumed in overcoming train-friction is—

$$\frac{3600}{85} \times 2{\cdot}10 = 89 \text{ w hr per ton.}$$

The energy consumption in w hr per hour may, of course, be expressed in terms of power, *i.e.* of the *rate* of consumption of energy. Thus—

$$1 \text{ w hr per hour} = 1 \text{ watt.}$$

We consequently arrive at the result that the average rate of consumption of electricity for overcoming the friction of the train is, taken over the entire route—

$$89 \text{ watts per ton.}$$

We have seen (on p. 30) that, taken over the entire route, the average rate of consumption of energy for the purpose of providing momentum is—

$$740 \text{ watts per ton.}$$

Thus, still retaining our assumption of 100 per cent. efficiency of the electrical equipment, we find that the average rate of consumption of electricity by the train is—

$$740 + 89 = 829 \text{ watts per ton.}$$

Determination of the Amount of Energy wasted as Heat at the Brakes

During drifting, and braking the train covers, as we find from Table VIII., on p. 28—

$$0{\cdot}166 + 0{\cdot}054 = 0{\cdot}22 \text{ mile.}$$

The energy required to overcome friction over this distance is—

$$26{\cdot}2 \times 0{\cdot}22 = 5{\cdot}8 \text{ w hr per ton.}$$

Thus every 85 seconds, the energy of momentum is drawn upon to the extent of 5·8 w hr per ton to overcome train-friction, and in the course of one hour the total amount of momentum employed in overcoming train-friction is—

$$\frac{3600}{85} \times 5{\cdot}8 = 250 \text{ w hr per ton.}$$

Consequently, the average rate, taken over the whole route, at which train-friction is being provided from the train's momentum is 250 watts per ton. We have already seen that the average rate at which energy is being accumulated in the form of momentum is—

$$740 \text{ watts per ton.}$$

Other than in overcoming train-friction, the momentum is ultimately transformed into heat at the brake-shoes. The rate, taken over the entire route, at which energy is being transformed into heat at the brake-shoes is—

$$740 - 250 = 490 \text{ watts per ton.}$$

Thus, in the present case, the energy ultimately wasted as heat at the brake-shoes constitutes—

$$\frac{490}{740} \times 100 = 66{\cdot}3 \text{ per cent.}$$

of the energy present in the train as momentum at the instant of attaining crest speed.

The average value of the power employed for train-friction, taken over the entire route, and including that taken directly from the line and that drawn from the momentum, is equal to—

$$829 - 490 = 339 \text{ watts per ton.}$$

Obviously, the same result is arrived at by adding 89 watts per ton, the average of the amount taken directly from the line as electricity, and 250 watts per ton, the amount taken from the stored-up energy of motion (*i.e.* the momentum) of the train. Thus we have also—

$$89 + 250 = 339 \text{ watts per ton.}$$

Modification of Calculations to take into Account the Losses in the Electrical Equipment on the Train

The train requires per ton an input from the line, at an average rate equal to 829 watts, *plus the losses in the electrical equipment.* For such a case as that which we are considering, we may (with sufficient accuracy, at any rate, for the purposes of this example) take the over-all efficiency of the electrical equipment on the train, averaged

over the whole route (*i.e.* including stops) at 72 per cent. Consequently, the average input to the train is—

$$\frac{829}{0{\cdot}72} = 1150 \text{ watts per ton.}$$

1150 watts per ton is, of course, 1150 w hr per ton per hour. Since the train covers 12·7 miles in one hour, the average rate of input of energy to the train may also be expressed as—

$$\frac{1150}{12{\cdot}7} = 90{\cdot}5 \text{ w hr per ton-mile.}$$

We have seen that the average rate at which energy is being transformed into momentum is 740 watts per ton. This is—

$$\frac{740}{1150} \times 100 = 64{\cdot}3 \text{ per cent.}$$

of the total input from the line to the train.

The average rate at which energy is devoted to the purposes of actual propulsion is—

$$\frac{339}{1150} \times 100 = 29{\cdot}5 \text{ per cent.}$$

of the total input from the line to the train.

Thus, we may say that, for this schedule, the "propulsive efficiency" is 29·5 per cent. By this we mean that, could we eliminate the losses in the electrical equipment and at the brake-shoes, only 29·5 per cent. as much energy would be required to operate the train as is, owing to these losses, actually required.

We can summarize these results as indicated in Table XI.—

TABLE XI.—ALLOCATION OF POWER CONSUMED BY A TRAIN DURING A GIVEN RUN.

	Average power required (watts).	Ditto, in per cent., of total power required.
Average rate at which energy is transformed into heat at the brake-shoes, per ton weight of train	490	42·6
Ditto into heat associated with useful propulsion, per ton weight of train	339	29·5
Ditto into heat in the electrical equipment, per ton weight of train (1150 − 490 − 339 =) . .	321	27·9
Total input to train per ton of weight . .	1150	100

From these results we find ample justification for the statement that, for the more usual and appropriate cases of railway electrification, a leading occurrence relates to imparting momentum to the train, and subsequently transforming this momentum into heat at

the brake-shoes. In the example we have taken, the average power consumption of the train is 1150 watts per ton. If we had assumed zero train-friction, still taking the average efficiency of the electrical equipment as 72 per cent., the average power consumption of the train for the entire run would have been—

$$\frac{740}{0{\cdot}72} = 1030 \text{ watts per ton.}$$

If, furthermore, we had assumed 100 per cent. efficiency of the electrical equipment, then the average power consumption of the train, taken over the entire run, would have been—

$$\frac{740}{1{\cdot}00} = 740 \text{ watts per ton.}$$

Reverting again from "average power consumption" to "energy consumption per ton-mile," we have the following results for the example we have taken—

(*a*) Energy consumed by the train on the assumption of a train-friction of 6 kg per ton = 90·5 w hr per ton-mile.

(*b*) Ditto, on the assumption of 100 per cent. efficiency of electrical equipment

$$= \frac{72}{100} \times 90{\cdot}5 = 79{\cdot}6 \text{ w hr per ton-mile.}$$

(*c*) Ditto, on the further assumption of zero train-friction—

$$\frac{0{\cdot}0278 \times 1{\cdot}09 \times 24^2}{0{\cdot}30} = 58{\cdot}1 \text{ w hr per ton-mile.}$$

The readily-obtained values set forth in Table XII. are also of interest—

TABLE XII.—ALLOCATION OF ENERGY CONSUMED BY A TRAIN DURING A GIVEN RUN.

Ultimate allocation of the energy.	W hr per ton-mile.	Per cent. of total input.
Energy transformed into heat at the brake-shoes $\left(\frac{490}{12{\cdot}7} =\right)$	38·6	42·6
Ditto into heat associated with useful propulsion, *i.e.* into train-friction $\left(\frac{339}{12{\cdot}7} =\right)$ . . .	26·7	29·5
Ditto into heat in the electrical equipment $\left(\frac{321}{12{\cdot}7} =\right)$	25·2	27·9
Total	90·5	100

The energy present as momentum at the instant of attaining crest speed is 58·1 w hr per ton-mile, which is $\left(\frac{58{\cdot}1}{90{\cdot}5} \times 100 = \right)$64·3 per cent. of the input to the train. This last value, and the three values in Table XII., are represented graphically in Fig. 20.

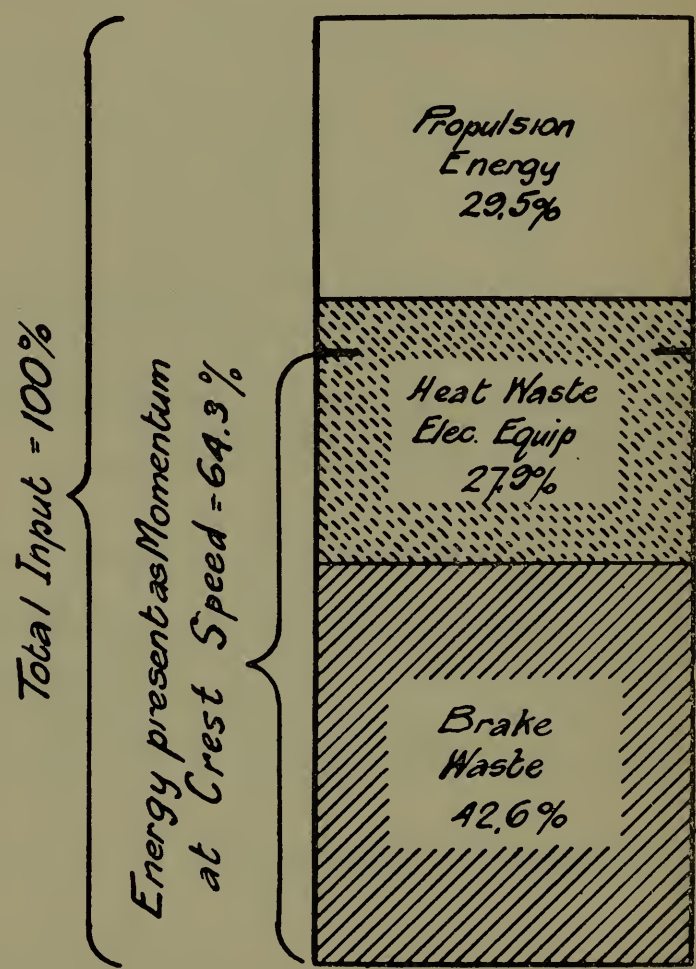
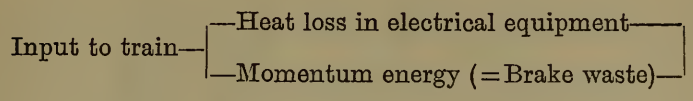


Fig. 20.—Diagram showing the Allocation of the Energy Input for a Service with an Average Distance of 0·30 Mile from Start to Stop, and a Schedule Speed of 12·7 ml ph (see Table XII., on p. 37).

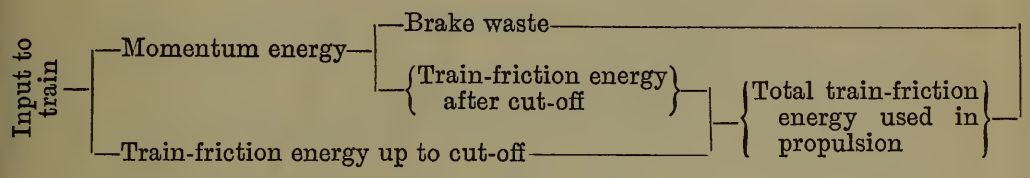
In considering the case of zero train-friction and 100 per cent. efficiency of the electrical equipment, the energy distribution is as follows:—

Input to train = Momentum energy = Brake waste.

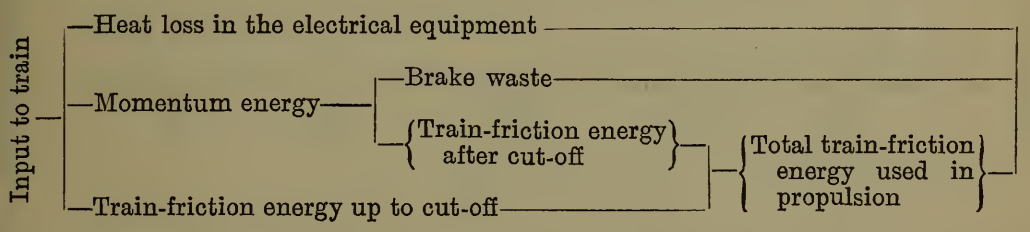
Allowing for losses in the electrical equipment, but retaining the assumption of zero train-friction, the distribution becomes—



If 100 per cent. efficiency of the electrical equipment is again assumed, but allowance is made for train-friction, then the distribution is shown by—



Finally, we have the general case of actual practice—



Calculations similar to the example in this chapter have been carried out for runs of 0·5, 1, 2, and 4 miles at schedule speeds of 15, 20, 25, and 30 ml ph. The results obtained are embodied in the curves of Figs. 21, 22, 23 and 24, which show, for these schedules, how the total input is distributed.

Fig. 25 is diagrammatic, and brings out the great contrast in the allocation of the energy for widely differing schedules.

When comparison is made with the results of practical tests, variations will, of course, be found in all cases, since it is impossible to estimate with complete precision the energy input for a given case; it varies greatly with profile of the track, the conditions of rail service, the design of the rolling stock, the type of equipment, the methods of motor control, and even the qualifications of the driver. The subject has been discussed from the standpoint employed in this chapter for the purpose of logically paving the way for the more useful (although somewhat less obvious) methods which will be set forth in the immediately following chapters.

In examining Figs. 21 to 25 special attention should be given to the influence of the schedule speed and the distance between stops. For a given schedule speed, then, the greater the distance between stops, the less is the percentage of energy wasted at the brakes, and

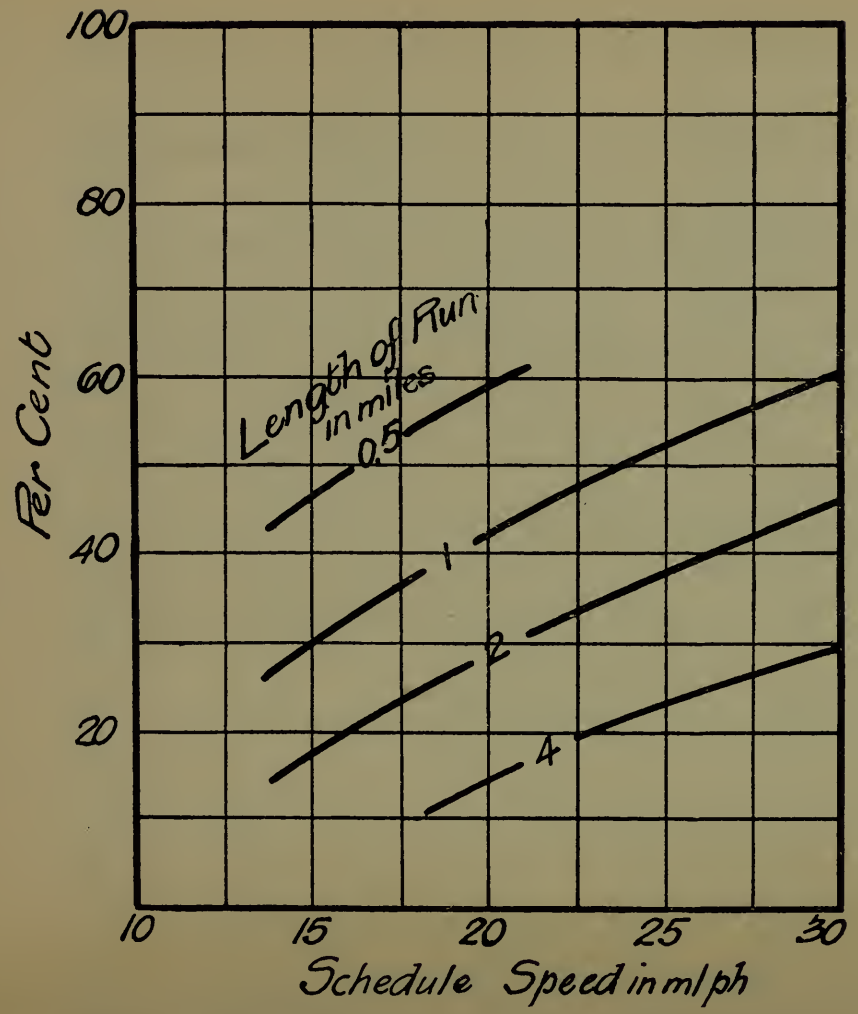


FIG. 21.—Energy of Momentum at Crest Speed as Percentage of Total Input.

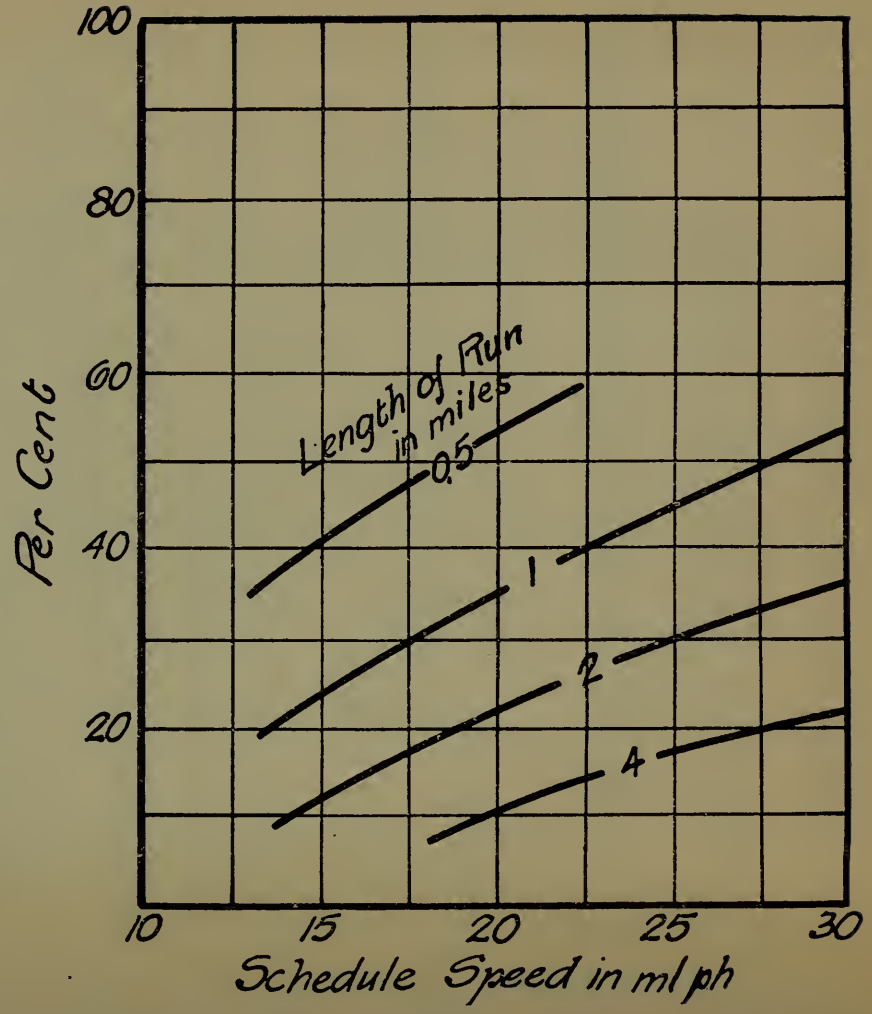


FIG. 22.—Waste in Heat at Brake-shoes as Percentage of Total Input.

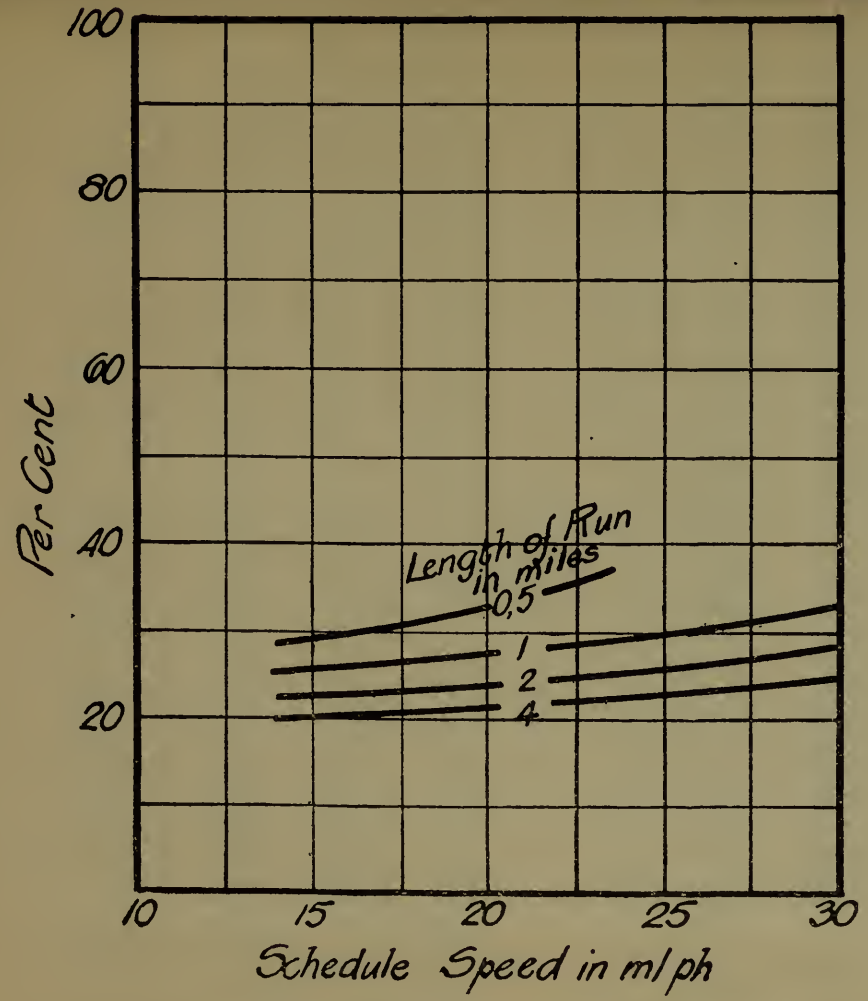


FIG. 23.—Waste as Heat in Electrical Equipment as Percentage of Total Input.

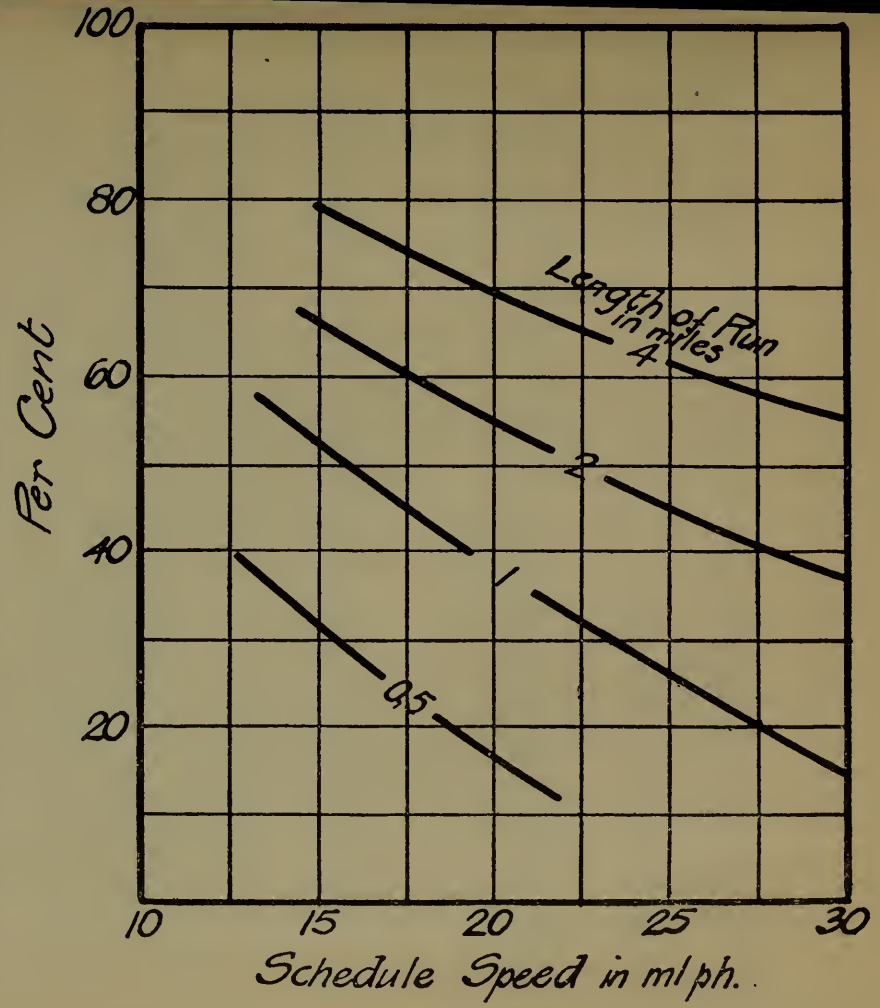


FIG. 24.—Propulsion Energy as Percentage of Total Input.

the greater is the percentage which may be considered as propulsion energy. Conversely, with a given distance between stops, the propulsion energy is a greater percentage of the total input, the lower the schedule speed. Fig. 23 shows rather strikingly the relatively

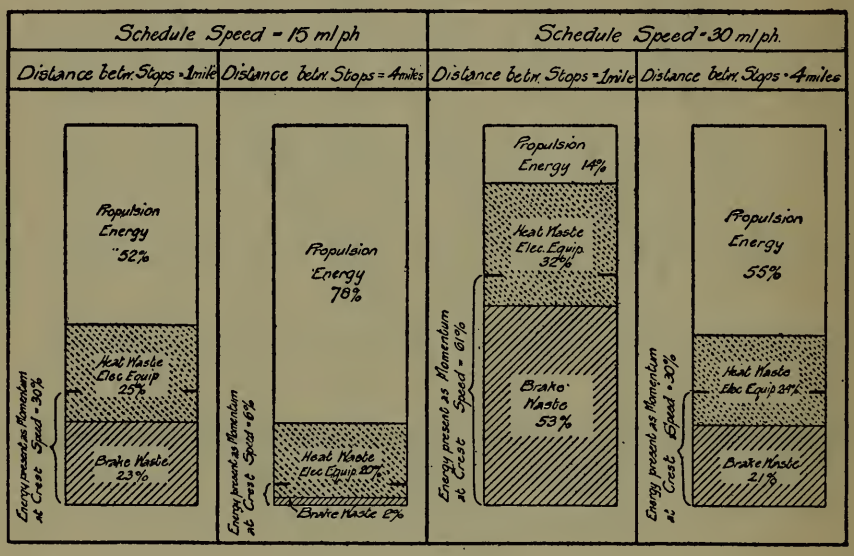


FIG. 25.—Diagrams showing Allotment of Total Input to 100-ton Train for Different Services, taken from the Curves of Figs. 21 to 24, and representing Actual Runs.

small percentages of the energy which is wasted in the electrical equipment, and the comparative independence of this percentage on the schedule speed, and even on the distance between stops.

EXAMPLES.

1. What is the amount of energy contained in a 50-ton train running at a speed of 20 ml ph (including the energy due to rotating parts, which may be taken as 9 per cent.)? *Ans.* 605 w hr.

2. Neglecting track friction, what amount of energy must be expended to accelerate a 75-ton train from 25 ml ph to 40 ml ph? *Ans.* 2·22 kw hr.

3. 1 w hr = 367 kg m. What distance would a train run on straight level track against track friction (of 3 kg per ton) by virtue of its energy of momentum at 15 ml ph? *Ans.* 832 m.

4. A train running at 30 ml ph mounts an incline of 1 in 100. How far will it run up the incline before its momentum energy is consumed, neglecting track friction? *Ans.* 1000 m.

5. What would have been the distance run in question 4 if the track friction were 4 kg per ton? *Ans.* 715 m.

6. A train weighing 150 tons and travelling at 51 ml ph is brought to rest in 30 seconds. Assuming no track friction, determine: (*a*) The decelerating rate.

(*b*) The energy wasted at the brake-shoes in kw hr. (*c*) The average rate at which energy, expressed in watts per ton weight of train, is being wasted at the brake-shoes. *Ans.* (*a*) 1·7 ml phps.
(*b*) 11·8 kw hr.
(*c*) 9450 watts per ton.

7. A train absorbed energy at the axles at the average rate of 300 kw for 40 seconds, and accelerated during this time at 1·2 ml phps. Assuming no friction, determine the weight of the train. *Ans.* 48 tons.

8. From Table IX., (*a*) What would be the tractive force of a locomotive hauling a train of five 20-ton coaches at a constant speed of 40 ml ph on straight, level track, and (*b*) How much energy is needed to propel the coaches over a distance of 4 miles?

Ans. (*a*) 4700 kg.
(*b*) 8·2 kw hr.

9. A hypothetical train is accelerated at a steady rate of 1·0 ml phps up to a crest speed of 50 ml ph, and the train-friction averages 6 kg per ton. Find the components of (*a*) the energy input for friction, and (*b*) the energy input for momentum. *Ans.* (*a*) Friction—9·1 w hr per ton.
(*b*) Momentum—75·9 w hr per ton.

10. (*a*) What is the distance covered up to crest speed in question 9? (*b*) Compare the energy input during the accelerating period in question 9 with the input necessary to propel the train the same distance at the constant speed of 50 ml ph (see Table XX.). *Ans.* (*a*) Distance—558 m.
(*b*) Accelerating period—85 w hr per ton.
Constant speed—10 w hr per ton.

11. The average rate of input to twenty 100-ton trains operating to a schedule speed of 20 ml ph is 2000 kw. Find the average input to the trains in w hr per ton-mile. *Ans.* 50 w hr per ton-mile.

12. If the propulsive efficiency is 30 per cent. in question 11, find the average input for overcoming friction. *Ans.* 300 watts per ton.

13. A three-coach train weighing 100 tons has a total of 12 axles and 24 wheels, each wheel being 1000 mm diameter and weighing 250 kg. The train is driven by 4 motors, each of the armatures of which weigh 1200 kg and measure 400 mm in diameter. If the gear ratio is 3·7, find the weight which must be added to the weight of train to allow for the rotational inertia of the armatures and wheels. Employ Carter's formula in footnote of page 30. Express the weight as a percentage of the total train weight. *Ans.* 8·9 per cent.

CHAPTER IV

A METHOD OF ESTIMATING THE ENERGY CONSUMPTION OF TRAINS, ON THE ASSUMPTION OF NEGLIGIBLE TRAIN-FRICTION AND OF ONE HUNDRED PER CENT. EFFICIENCY OF THE ELECTRICAL EQUIPMENT ON THE TRAIN

THE investigations in the preceding chapter have shown that when trains are operated at high schedule speeds, and with frequent stops, a large percentage of the total energy consumed by the train is required in providing the momentum corresponding to the crest speed (*i.e.* the maximum speed attained at any time during the run), and that a large part of this energy of momentum is subsequently transformed into heat at the brake-shoes.

In order to bring clearly into prominence the significance of this circumstance as affecting the question of the electric operation of city and suburban railways, it is very instructive to consider, in the first instance, the case of a train assumed to have no friction loss (other than at the brake-shoes), and no loss in the electrical equipment. Thus, the only energy consumed by the train is that transformed into the momentum of the train corresponding to its crest speed.

Let us first work out several cases where the acceleration from rest, up to the crest speed, is of the uniform value of 1 ml phps. When the crest speed is reached, the supply of electricity is cut off, and the train runs at constant speed (since we have assumed that there is no train-friction) until the brakes are applied. The brake pressure is assumed to be adjusted at such a value that the deceleration is 1·5 ml phps. The shape of the speed-time diagram corresponding to these conditions is shown in Fig. 26. With these rates of acceleration and deceleration, let us first work through a case where the schedule speed is 18 ml ph, and where the train makes one 20-second stop per mile.

In one hour there will be 18 stops. Therefore, the total time during which the train is at rest during each hour is equal to –

$$18 \times 20 = 360 \text{ seconds.}$$

Consequently, the train is in motion for—

$$3600 - 360 = 3240 \text{ seconds per hour,}$$

and the time occupied by one run from start to stop is equal to—

$$\frac{3240}{18} = 180 \text{ seconds.}$$

Let us denote by F the crest speed (*i.e.* the maximum speed attained by the train at any time during the run) in ml ph. Then the

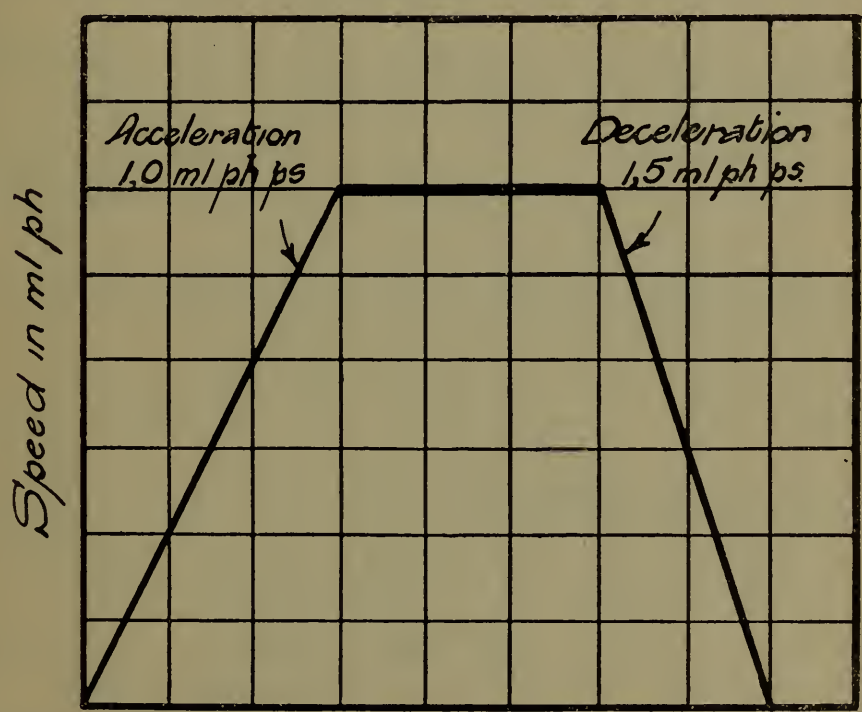


FIG. 26.—Hypothetical Speed-time Diagram, with Acceleration and Deceleration of 1·0 and 1·5 ml phps respectively, and no Track Friction.

number of seconds occupied in accelerating the train at 1·0 ml phps is equal to—

$$\frac{F}{1{\cdot}0} = F$$

Since the deceleration is at 1·5 ml phps as against an acceleration of 1·0 ml phps the number of seconds during which the train is being braked is equal to

$$\frac{F}{1{\cdot}5}$$

The number of seconds during which the train is running at constant speed is equal to—

$$180 - F - \frac{F}{1{\cdot}5}.$$

We thus have the values in Table XIII.—

TABLE XIII.—SHOWING THE DURATION OF, AND THE AVERAGE SPEED AND DISTANCE COVERED DURING, THE ACCELERATING, CONSTANT SPEED AND DECELERATING PERIODS OF THE SPEED-TIME DIAGRAM OF FIG. 26, WHEN TAKEN TO REPRESENT A 1-MILE RUN AT A SCHEDULE SPEED OF 18 ML PH.

—	Duration (seconds).	Duration (hours).	Average speed (ml ph).	Distance covered (miles).
Accelerating period	F	$\frac{1}{3600}\times F$	$\frac{F}{2}$	$\frac{1}{3600}\times F\times\frac{F}{2}$
Constant speed period	$180-F-\frac{F}{1\cdot5}$	$\frac{1}{3600}\left(180-F-\frac{F}{1\cdot5}\right)$	F	$\frac{1}{3600}\left(180-F-\frac{F}{1\cdot5}\right)F$
Decelerating period	$\frac{F}{1\cdot5}$	$\frac{1}{3600}\times\frac{F}{1\cdot5}$	$\frac{F}{2}$	$\frac{1}{3600}\times\frac{F}{1\cdot5}\times\frac{F}{2}$

The *total* distance traversed is one mile; therefore, by adding the distances covered during the three periods, we have—

$$\frac{1}{3600}\left[F\times\frac{F}{2}+\left(180-F-\frac{F}{1\cdot5}\right)F+\frac{F}{1\cdot5}\times\frac{F}{2}\right]=1$$

$$\frac{1}{3600}\left(\frac{F^2}{2}+180F-\frac{5}{3}F^2+\frac{F^2}{3}\right)=1$$

$$\frac{5}{6}F^2-180F=-3600$$

$$F^2-216F=-4330$$

$$F^2-216F+11{,}670=-4330+11{,}670$$

$$F-108=\pm85\cdot7$$

$$F=22\cdot3$$

$\therefore$ The crest speed is equal to 22·3 ml ph.

Duration of accelerating period $=\frac{22\cdot3}{1\cdot0}=22\cdot3$ seconds.

Duration of constant speed period

$$=180-22\cdot3-\frac{22\cdot3}{1\cdot5}=142\cdot8 \text{ seconds.}$$

Duration of braking period $=\frac{22\cdot3}{1\cdot5}=14\cdot9$ seconds.

Since we have assumed that the friction is negligible, we have only to consider the energy required to give the necessary momentum at this crest speed.

Energy of translational momentum per ton is equal to—

$$0\cdot0278\times(22\cdot3)^2=13\cdot8 \text{ w hr.}$$

The energy of rotational momentum of the revolving parts, consisting of the wheels of the trucks and the armatures of the motors, will, in most cases, increase the above amount by some 9 per cent. Consequently, we have—

Energy of translational and rotational momentum per ton is equal to—

$$\begin{aligned} & 1{\cdot}09 \times 0{\cdot}0278 \times (22{\cdot}3)^2 \\ &= 0{\cdot}0303 \times (22{\cdot}3)^2 \\ &= 15{\cdot}0 \text{ w hr.} \end{aligned}$$

We have carried out, step by step, the calculations for this first case. It will now, however, be of interest to develop a formula by means of which the calculations of such cases may be considerably expedited. Let us, as before, denote the crest speed by F.

Let A equal the acceleration in ml phps.
„ B „ „ deceleration during braking in ml phps.
„ T „ „ time in seconds occupied by a single run from start to stop.
„ M „ „ distance in miles from start to stop.
„ F „ „ crest speed in ml ph.

We first obtain the values in Table XIV.—

TABLE XIV.—SHOWING THE DURATION OF, AND THE AVERAGE SPEED AND DISTANCE COVERED DURING, THE ACCELERATING, CONSTANT SPEED AND DECELERATING PERIODS OF A REPRESENTATIVE SPEED-TIME DIAGRAM.

—	Duration (seconds).	Duration (hours).	Average speed (ml ph).	Distance covered (mile).
Accelerating period .	$\frac{F}{A}$	$\frac{1}{3600} \times \frac{F}{A}$	$\frac{F}{2}$	$\frac{1}{3600} \times \frac{F}{A} \times \frac{F}{2}$
Constant speed period .	$T - \frac{F}{A} - \frac{F}{B}$	$\frac{1}{3600}\left(T - \frac{F}{A} - \frac{F}{B}\right)$	F	$\frac{1}{3600}\left(T - \frac{F}{A} - \frac{F}{B}\right)F$
Decelerating period .	$\frac{F}{B}$	$\frac{1}{3600} \times \frac{F}{B}$	$\frac{F}{2}$	$\frac{1}{3600} \times \frac{F}{B} \times \frac{F}{2}$

Total distance covered in miles is equal to M.

$$\therefore \frac{1}{3600}\left(\frac{F^2}{2A} + TF - \frac{F^2}{A} - \frac{F^2}{B} + \frac{F^2}{2B}\right) = M$$

$$\frac{F^2}{2A} + \frac{F^2}{2B} - TF = -M \times 3600$$

$$\frac{A + B}{2AB} \times F^2 - TF = -M \times 3600$$

$$\text{Let} \quad \frac{AB}{A + B} = C$$

$$\therefore \; F^2 - 2C \times T \times F = -7200 \times C \times M$$

$$\text{and} \quad F = \frac{2C \times T \pm 2\sqrt{C^2 \times T^2 - 7200 \times C \times M}}{2}$$

Thus, for F, the Crest Speed, we have the formula—

$$F = C \times T - \sqrt{C^2 \times T^2 - 7200 \times C \times M}$$

since the positive value before the root will be always inadmissible.

Values of C are found (for any particular acceleration and deceleration) from Table XV.—

TABLE XV.—VALUES OF C FOR VARIOUS ACCELERATIONS AND DECELERATIONS.

A, the Acceleration (ml phps).	The factor C, in the crest-speed formula, for the following values of B, the braking deceleration (ml phps).										
	B=1·0.	B=1·1.	B=1·2.	B=1·3.	B=1·4.	B=1·5.	B=1·6.	B=1·7.	B=1·8.	B=1·9.	B=2·0.
0·5	0·333	0·344	0·353	0·361	0·368	0·375	0·381	0·386	0·391	0·396	0·40
0·6	0·375	0·388	0·40	0·411	0·420	0·429	0·436	0·443	0·450	0·456	0·462
0·7	0·412	0·428	0·442	0·455	0·467	0·478	0·487	0·496	0·504	0·512	0·519
0·8	0·444	0·463	0·480	0·495	0·509	0·522	0·533	0·544	0·554	0·563	0·571
0·9	0·474	0·495	0·514	0·532	0·548	0·563	0·576	0·588	0·60	0·611	0·621
1·0	0·50	0·524	0·545	0·565	0·583	0·60	0·615	0·630	0·643	0·655	0·667
1·1	0·524	0·550	0·574	0·596	0·616	0·634	0·652	0·668	0·683	0·697	0·710
1·2	0·545	0·574	0·60	0·624	0·646	0·667	0·686	0·703	0·720	0·735	0·750
1·3	0·565	0·596	0·624	0·650	0·674	0·696	0·717	0·737	0·755	0·772	0·788
1·4	0·583	0·616	0·646	0·674	0·70	0·724	0·747	0·768	0·787	0·806	0·824
1·5	0·60	0·635	0·667	0·696	0·724	0·750	0·774	0·797	0·818	0·838	0·857
1·6	0·615	0·652	0·686	0·717	0·747	0·774	0·80	0·824	0·847	0·869	0·889
1·7	0·630	0·668	0·703	0·737	0·768	0·797	0·824	0·850	0·874	0·897	0·919
1·8	0·643	0·683	0·720	0·755	0·787	0·818	0·847	0·874	0·90	0·924	0·947
1·9	0·655	0·697	0·735	0·772	0·806	0·838	0·869	0·897	0·924	0·950	0·974
2·0	0·667	0·710	0·750	0·788	0·824	0·857	0·889	0·919	0·947	0·974	1·00

By means of the above formula for the crest speed, and of Table XV., we can quickly carry through calculations, on the assumptions employed in this chapter, for the energy consumed at the train per ton-mile.

Let us first take the case of a route where the distance between stations is 0·5 mile. Let the acceleration be 1·0 ml phps and the deceleration during braking 1·5 ml phps, *i.e.*—

$$M = 0{\cdot}5 \qquad A = 1{\cdot}0 \qquad B = 1{\cdot}5.$$

For these conditions let us estimate the consumption in watt-hours per ton-mile for speeds of 9, 12, 15 and 18 ml ph. Let the duration of each stop be 20 seconds in all cases, *i.e.* let Q = 20.

We may carry out the calculations in orderly form, as in the first part of Table XVI., which also shows similar calculations for 1-, 2- and 4-mile runs with the same acceleration and deceleration. Considering the results for a 0·5-mile run, in *doubling* the *schedule* speed (from 9 ml ph to 18 ml ph) we have considerably more than *trebled* the *crest* speed; and the values in the last horizontal line of these calculations show that for the higher schedule speed (18 ml ph) the value of the w hr per ton-mile is eleven times as great as for the lower schedule speed (9 ml ph).

In Fig. 27 are plotted, for various values of M (the distance in

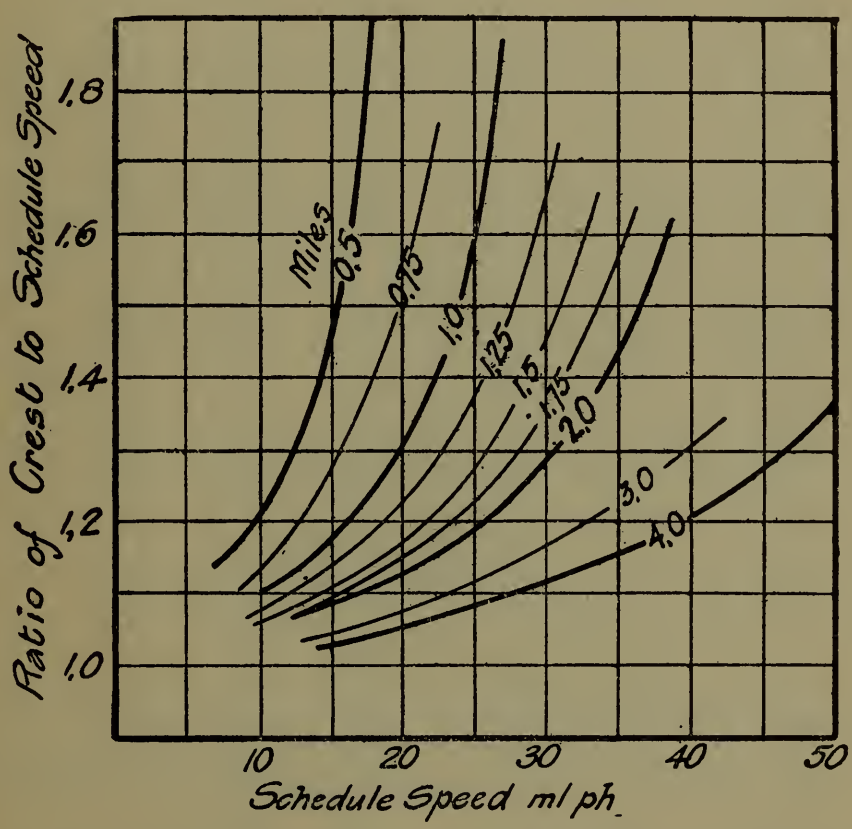


FIG. 27.—Ratio of Crest to Schedule Speed for Various Schedule Speeds and Runs; with Acceleration of 1·0 ml phps, Braking 1·5 ml phps. (Neglecting Deceleration during Coasting, *i.e.* assuming Frictionless Run.)

miles from start to stop), the corresponding ratios of the crest speed to the schedule speed.

The shape of the curves indicates the limiting values for the attainable speeds for these values of the acceleration and deceleration. Thus, it is evident that with a distance of only 0·5 mile from start to stop, and with these values of the acceleration and deceleration, a

TABLE XVI.—CALCULATION OF THE ENERGY CONSUMPTION OF TRAINS FOR ASSUMED FRICTIONLESS RUNS.

(Acceleration, 1·0 ml phps. Deceleration during braking = 1·5 ml phps. Efficiency of electrical equipment assumed as 100 per cent.)

Distance between stops (mile) [M]	0·5				1					2					4				
Schedule speed (ml ph) .	9	12	15	18	12	15	21	24	27	18	24	27	30	36	24	30	36	42	48
Number of stops per hour	18	24	30	36	12	15	21	24	27	9	12	13·5	15	18	6	7·5	9	10·5	12
Duration of each stop (seconds) [Q]	20				20					20					20				
Total number of seconds in each hour during which the train is at rest	360	480	600	720	240	300	420	480	540	180	240	270	300	360	120	150	180	210	240
Total number of seconds in each hour during which the train is in motion	3,240	3,120	3,000	2,880	3,360	3,300	3,180	3,120	3,060	3,420	3,360	3,330	3,300	3,240	3,480	3,450	3,420	3,390	3,360
Number of seconds from start to stop [T]	180	130	100	80	280	220	151	130	113	380	280	246	220	180	580	460	380	328	280
Factor C from Table XV.	0·6				0·6					0·6					0·6				
CT	108	78	60	48	168	132	90	78	68	228	168	148	132	108	348	276	228	194	168
C^2T^2	11,670	6,084	3,600	2,304	28,220	17,420	8,100	6,084	4,624	51,980	28,220	21,900	17,420	11,670	121,100	76,080	51,980	37,640	28,220
7200 CM	2,160				4,320					8,640					17,280				
$C^2T^2 - 7200$ CM . .	9,510	3,924	1,440	144	23,900	13,100	3,780	1,764	294	43,340	19,580	13,260	8,780	3,030	103,820	58,800	34,700	20,360	10,940
$\sqrt{C^2T^2 - 7200\ CM}$ . .	97·5	62·7	38	12	154·5	114·4	61·5	42	17·1	208	139·9	115·1	93·7	55	322	242·5	186·2	142·6	104·7
Crest speed, $F = CT - \sqrt{C^2T^2 - 7200\ CM}$	10·5	15·3	22·0	36·0	13·5	17·6	28·5	36·0	50·9	20·0	28·1	32·9	38·3	53·0	26·0	33·5	41·8	51·4	63·3
Ratio of crest to schedule speed	1·17	1·28	1·47	2·00	1·12	1·17	1·36	1·50	1·88	1·11	1·17	1·22	1·28	1·47	1·08	1·12	1·16	1·23	1·32
Momentum of train at speed F in w hr per ton ($= 0·0303F^2$)	3·4	7·1	14·7	39·3	5·5	9·4	24·6	39·3	78·5	12·1	23·9	32·8	44·4	85	20·5	34·0	53·0	80·0	121
Number of stops per mile .	2·0				1·0					0·5					0·25				
Train consumption in w hr per ton-mile	6·7	14·2	29·4	78·6	5·5	9·4	24·6	39·3	78·5	6·1	12·0	16·4	22·2	42·5	5·1	8·5	13·3	20·0	30·3

schedule speed of 20 ml ph is unattainable, as also a speed of 30 ml ph for M = 1·0.

In Fig. 28 are shown curves, drawn for various values of M, with schedule speeds as abscissae and with the energy consumption in w hr per ton-mile as ordinates.

This last figure shows us that the energy consumption (for frictionless runs and 100 per cent. efficiency of equipment) increases

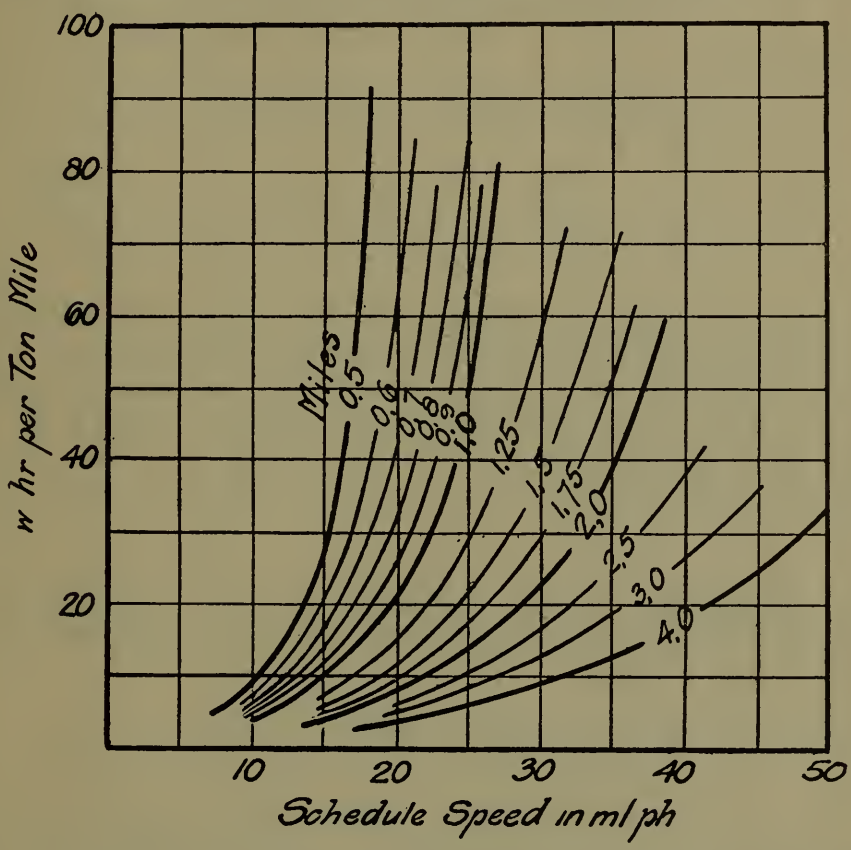


FIG. 28.—Energy Consumption for Various Schedule Speeds and Runs under the assumed Frictionless Conditions, and 100 per cent. Efficiency of Electrical Equipment. Acceleration 1·0 ml phps; Braking 1·5 ml phps.

at an exceedingly rapid rate (as the square, in fact, of the crest speed). From Fig. 27 we have already seen that the crest speed increases much more rapidly than the schedule speed. For these reasons, schedule speeds are soon reached which, while attainable, are not commercial, as the cost of the required energy would be prohibitive. The limiting schedule speed, when commercial considerations are taken into account, is, for short distances between stops, very much lower than is generally appreciated. In Table XVII. are given values of the schedule speeds attainable with various

values of the energy input for supplying momentum, for runs of 0·5, 1·0, and 2·0 miles from start to stop, the values being taken from Fig. 28.

TABLE XVII.—SCHEDULE SPEEDS ATTAINABLE WITH VARIOUS VALUES OF ENERGY INPUT, AND WITH DIFFERENT LENGTHS OF RUN, ASSUMING FRICTIONLESS RUNS AND 100 PER CENT. EFFICIENCY OF ELECTRICAL EQUIPMENT.

Distance from start to stop (miles).	Schedule speed in ml ph for the following values of the energy input to provide momentum (w hr per ton-mile).		
	40 w hr per ton-mile.	50 w hr per ton-mile.	60 w hr per ton-mile.
0·5	16·4	16·9	17·3
1·0	23·7	25·0	25·7
2·0	35·2	37·3	38·5

It must be distinctly remembered that the above values only apply to the values of the acceleration and deceleration (A = 1·0 and B = 1·5) employed in the calculations, and are also restricted to the assumptions made throughout this chapter, namely, that the train-friction is negligible and that the efficiency of the electrical equipment is 100 per cent.

The Influence of the Values of the Acceleration and Deceleration on the Energy Consumption at the Train

Let us take the case of a 0·5-mile run from start to stop. Thus, M = 0·5. Let the acceleration and deceleration be *infinite*, and let the train run at a constant speed of 20 ml ph from start to stop. Then the time occupied in covering the 0·5-mile distance from start to stop will be—

$$\frac{0{\cdot}5}{20} \times 3600 = 90 \text{ seconds.}$$

Taking the duration of the stop as 20 seconds, the schedule speed is—

$$\frac{90}{110} \times 20 = 16{\cdot}3 \text{ ml ph.}$$

Since the crest speed is 20 ml ph the translational momentum is—

$$0{\cdot}0278 \times 20^2 = 11{\cdot}1 \text{ w hr per ton.}$$

Thus, after allowing for the rotational momentum, we find that the *total* momentum is—

$$11{\cdot}1 \times 1{\cdot}09 = 12{\cdot}1 \text{ w hr per ton.}$$

Still assuming that the train friction is negligible, and that the efficiency of the electrical equipment is 100 per cent., we have a consumption of—

$$2 \times 12{\cdot}1 = 24{\cdot}2 \text{ w hr per ton-}\textit{mile}.$$

Now let us go to the other limit, *i.e.* to the *lowest* rate of accelerating and braking which will permit of obtaining the average speed of 20 ml ph for a 0·5-mile run. The speed-time diagram will be as in Fig. 29.

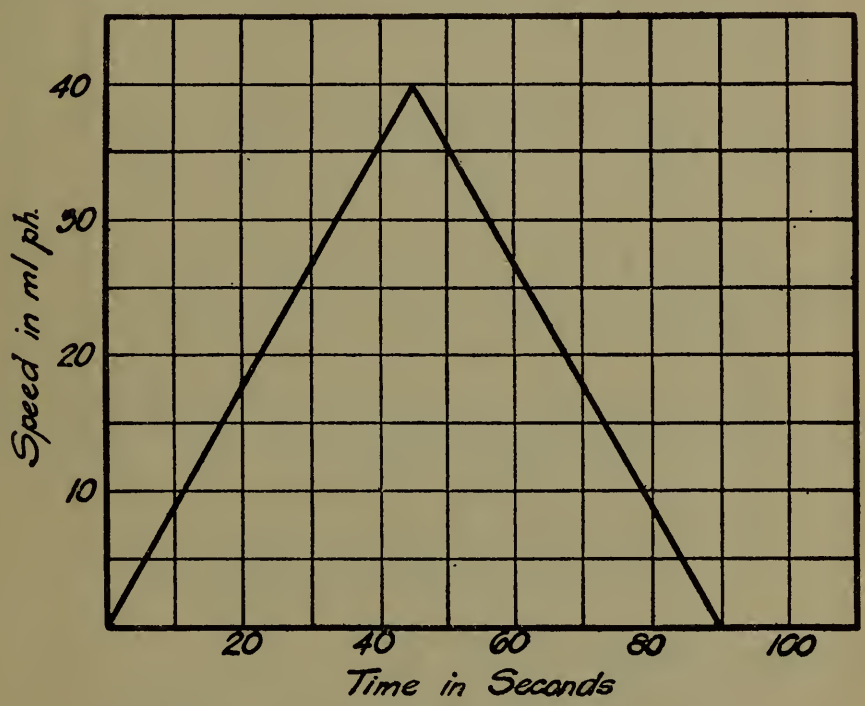


FIG. 29.—Speed-time Diagram for 0·5-mile Run under Limiting Conditions. No Coasting Period and Minimum Acceleration and Declaration.

The crest speed will be 40 ml ph, and will be attained in—

$$\frac{90}{2} = 45 \text{ seconds.}$$

∴ The acceleration is at the rate of—

$$\frac{40}{45} = 0{\cdot}89 \text{ ml phps.}$$

The translational momentum at the crest speed will be—

$$0{\cdot}0278 \times 40^2 = 44{\cdot}5 \text{ w hr per ton.}$$

The total momentum $= 44{\cdot}5 \times 1{\cdot}09 = 48{\cdot}5$ w hr per ton. Neglecting

train-friction, and assuming 100 per cent. efficiency of the electrical equipment, the consumption at the train will be—

$$2 \times 48{\cdot}5 = 97 \text{ w hr per ton-mile.}$$

Now let us take an intermediate case with the acceleration and deceleration both equal to 1·5 ml phps. The diagram is shown in Fig. 30.

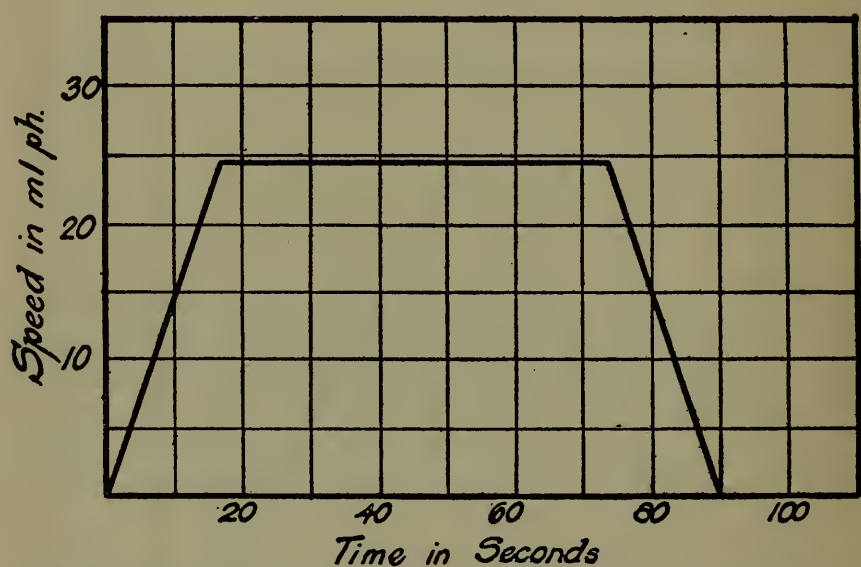


FIG. 30.—Speed-time Diagram for 0·5-mile Run with no Track Friction. Acceleration and Braking of 1·5 ml phps.

We have the formula—

$$\text{Crest speed} = F = CT - \sqrt{C^2T^2 - 7200\,CM}$$

and from Table XV. we obtain—

$$C = 0{\cdot}75$$
$$\therefore F = 0{\cdot}75 \times 90 - \sqrt{0{\cdot}563 \times 8100 - 7200 \times 0{\cdot}75 \times 0{\cdot}5}$$
$$= 24{\cdot}5 \text{ ml ph.}$$

Translational momentum $= 0{\cdot}0278 \times 24{\cdot}5^2 = 16{\cdot}7$ w hr per ton
Total momentum $= 1{\cdot}09 \times 16{\cdot}7 = 18{\cdot}2$ „ „
$= 36{\cdot}4$ w hr per ton-mile.

Let us take one more case, employing the acceleration and deceleration of 1·0 ml phps.

From this value we have, from Table XV.—

$$C = 0{\cdot}5$$
$$F = 0{\cdot}5 \times 90 - \sqrt{0{\cdot}25 \times 8100 - 7200 \times 0{\cdot}5 \times 0{\cdot}5} = 30 \text{ ml ph.}$$

Translational momentum = $0{\cdot}0278 \times 30^2 = 25{\cdot}0$ w hr per ton
Total momentum = $1{\cdot}09 \times 25{\cdot}0 = 27{\cdot}2$ „ „
= 54·4 w hr per ton-mile.

Let us summarise these results as shown in Table XVIII.—

TABLE XVIII.—ENERGY CONSUMPTION FOR A 0·5-MILE RUN, WITH VARIOUS VALUES OF ACCELERATION AND DECELERATION (ASSUMING FRICTIONLESS RUNS AND ELECTRICAL EQUIPMENT OF 100 PER CENT. EFFICIENCY).

Acceleration and deceleration (ml phps).	Crest speed (ml ph).	Average speed (ml ph).	Schedule speed (ml ph).	Momentum energy at crest speed.	
				w hr per ton.	w hr per ton-mile.
0·89	40	20	16·3	44·5	97
1·00	30	20	16·3	27·2	54·4
1·50	24·5	20	16·3	18·2	36·4
Infinite	20	20	16·3	12·1	24·2

Thus we see that, so far as the amount of energy required for momentum is concerned, we ought to employ as high an acceleration and deceleration as practicable.

But, as we shall subsequently see, the employment of high accelerations imposes on the Electricity Supply Station, high peaks of load. As already stated, the commercially economical mean is, for city and suburban passenger trains, usually found to be some 1·0 to 1·5 ml phps (see p. 5).

From these considerations it is apparent that it is difficult to generalise as to the energy consumption at the train, corresponding to the maintenance of a given schedule speed, since the result depends very greatly indeed upon the acceleration and deceleration employed.

It is obvious that an additional objection to the use of low values of acceleration and deceleration is the high crest value of the speed thus rendered necessary in obtaining a given schedule speed.

A frequently recurring problem is, however, that relating to the lowest values of acceleration and deceleration which may be employed to obtain a given schedule speed for a given value of M, the distance from start to stop. Or, conversely, we may require to ascertain the highest schedule speed attainable, with accelerations and decelerations equal respectively, to stipulated values of A and B. To obtain this we may proceed as follows :—

As in our preceding investigations, we may denote by F the crest value of the speed in ml ph.

Then, for straight-line acceleration, we have—

$$\text{Average speed (ml ph)} = \frac{F}{2}$$

$$\text{Time occupied by acceleration (seconds)} = \frac{F}{A}$$

$$\text{,, ,, braking (seconds)} = \frac{F}{B}$$

∴ Distance covered, in miles, is equal to—

$$M = \frac{F}{3600 \times 2}\left(\frac{F}{A} + \frac{F}{B}\right) \quad \text{(See p. 47.)}$$

$$= \frac{F^2}{3600} \times \frac{A + B}{2AB}$$

$$\therefore \quad F^2 = 3600 \times \frac{2AB}{A + B} \times M$$

$$= 7200 \times C \times M$$

$$\text{and} \quad F = \sqrt{7200 \times C \times M}$$

F is the crest speed at the highest schedule speed corresponding to the stipulated conditions.

$$\text{Average speed (in ml ph)} = \frac{\sqrt{7200 \times C \times M}}{2}$$

$$\left.\begin{array}{c}\text{Time occupied by the entire}\\ \text{run from start to stop}\end{array}\right\} = T = \frac{F}{A} + \frac{F}{B} = F\left(\frac{A + B}{AB}\right)$$

$$\text{But} \quad \frac{AB}{A + B} = C$$

$$\therefore \quad T = \frac{F}{C}$$

$$\text{Also} \quad F = \sqrt{7200 \times C \times M}.$$

$$\therefore \quad T = \frac{\sqrt{7200 \times C \times M}}{C}$$

Now, the schedule speed $= \frac{T}{T + Q} \times$ average speed, where Q is the average duration of stop in seconds.

Therefore, by substitution, we have—

$$\left.\begin{array}{c}\text{Limiting}\\ \text{schedule speed}\\ \text{(ml ph)}\end{array}\right\} = \frac{\sqrt{7200 \times C \times M}}{C \times \left(\frac{\sqrt{7200 \times C \times M}}{C} + Q\right)} \times \frac{\sqrt{7200 \times C \times M}}{2}$$

$$= \frac{3600 \times C \times M}{\sqrt{7200 \times C \times M} + Q \times C}$$

; Values of S, t

	B = 1·2			B = 1·9		B = 2·0		
	F			F	W	S	F	W
·9	35·6		·6	37·7		15·7	37·9	
·1	50·4	7	·2	53·3	86·4	23·3	53·6	87·2
·4	71·3		·2	75·5		34·4	75·9	
·0	100·8		·7	106·8		50·0	107·4	
·7	37·9		·5	40·5		16·6	40·8	
·3	53·6	8	·8	57·3	99·5	24·9	57·6	100
·4	75·9		·4	81·1		36·6	81·6	
·0	107·4		·1	114·6		53·4	115·3	
·3	39·9		·3	42·9		17·4	43·2	
·4	56·4	9	·0	60·7	112	26·2	61·1	103
·9	79·8		·3	85·9		38·6	86·4	
·4	112·9		·0	121·5		56·4	122·2	
·9	41·5		·0	45·0		18·1	45·3	
·3	58·8	10	·1	63·7	123	27·2	64·2	124
·2	83·2		·0	90·4		40·3	90·7	
·4	43·0		·6	46·9		18·7	47·3	
·0	60·8	11	·0	66·3	133	28·2	66·9	135
·4	86·1		·5	93·8		41·8	94·6	
·8	44·3		·1	48·5		19·2	49·0	
·7	62·7	11	·9	68·6	143	29·0	69·3	145
·4	88·6		·8	97·1		43·0	98·0	
·1	45·4		·6	50·1		19·7	50·5	
·3	64·2	12	·6	70·8	152	29·8	71·5	155
·3	90·9		·0	100·1		44·4	101·1	
·5	46·5		·1	49·7		20·2	52·0	
·8	65·7	13	·3	72·8	160	30·5	73·5	163
·2	92·9		·0	103·0		45·4	104·0	
·8	47·4		·4	52·7		20·6	53·2	
·3	67·0	13	·9	74·6	168	31·1	75·3	172
·9	94·8		·9	105·4		46·3	106·8	
·0	48·2		·8	53·9		20·9	54·4	
·6	68·2	14	·4	76·2	176	31·7	77·0	179
·5	96·4		·9	107·9		47·4	108·9	
·2	49·0		·0	54·9		21·2	55·5	
·0	69·3	14	·9	77·7	183	32·2	78·6	187
·0	98·0		·7	109·8		48·2	111·1	
·5	49·7		·4	56·0		21·5	56·6	
·3	70·3	14	·4	79·0	189	32·7	80·0	194
·6	99·3		·4	112·0		48·9	113·2	
·7	50·3		·6	56·8		21·8	57·5	
·7	71·2	15	·9	80·4	195	33·2	81·3	200
·1	100·7		·1	113·6		49·6	115·0	
·9	50·9		·8	57·7		22·0	58·4	
·0	71·9	15	·2	81·6	201	33·6	82·6	206
·6	101·9		·9	115·7		50·4	116·8	
·1	49·7		·1	58·4		22·3	59·2	
·3	72·8	16	·6	82·7	207	34·0	83·8	212
·0	103·0		·3	116·9		50·9	118·0	
·2	52·0		·3	59·2		22·5	60·0	
·5	73·5	16	·0	83·8	212	34·4	84·8	218
·5	104·0		·9	118·0		51·5	120·0	

TABLE XIX.—LIMITING VALUES OF SPEED AND ENERGY CONSUMPTION, NEGLECTING TRAIN-FRICTION AND ASSUMING 100 PER CENT. EFFICIENCY OF ELECTRICAL EQUIPMENT, THE SPEED-TIME DIAGRAMS HAVING NO COASTING PERIOD (SEE FIGS. 29 AND 32).

A, the Acceleration (ml phps).	M, Length of Run (miles).	Limiting Values of S, the Schedule Speed in ml ph; F, the Crest Speed in ml ph; and of W, the Train Consumption in w hr per Ton-mile, for Different Values of A, B, and M.																																			
		B (the Deceleration during Braking) = 1·0			B = 1·1			B = 1·2			B = 1·3			B = 1·4			B = 1·5			B = 1·6			B = 1·7			B = 1·8			B = 1·9			B = 2·0		
		S	F	W	S	F	W	S	F	W	S	F	W	S	F	W	S	F	W	S	F	W	S	F	W	S	F	W	S	F	W	S	F	W	S	F	W
0·5	0·5	14·5	34·7	72·5	14·7	35·2	75·0	14·9	35·6	77·0	15·0	36·0	78·6	15·1	36·4	80·1	15·2	36·7	81·7	15·3	37·0	83·0	15·4	37·3	84·1	15·5	37·5	85·2	15·6	37·7	86·4	15·7	37·9	87·2			
	1	21·5	49·0		21·8	49·7		22·1	50·4		22·3	51·0		22·5	51·5		22·7	52·0		22·9	52·4		23·0	52·7		23·1	53·0		23·2	53·3		23·3	53·6			
	2	31·6	69·3		32·0	70·4		32·4	71·3		32·8	72·1		33·1	72·9		33·4	73·5		33·6	74·1		33·8	74·6		34·0	75·1		34·2	75·5		34·4	75·9			
	4	46·0	98·0		46·5	99·5		47·0	100·8		47·5	102·0		48·0	103·1		48·5	104·0		48·8	104·9		49·1	105·6		49·5	106·2		49·7	106·8		50·0	107·4			
0·6	0·5	15·2	36·7	81·6	15·5	37·4	84·5	15·7	37·9	87·2	15·9	38·4	89·6	16·0	38·9	91·5	16·1	39·3	93·5	16·2	39·6	95·1	16·3	39·9	96·5	16·4	40·2	98·0	16·5	40·5	99·5	16·6	40·8	100			
	1	22·7	52·0		23·0	52·9		23·3	53·6		23·6	54·3		23·8	55·0		24·0	55·6		24·2	56·1		24·4	56·5		24·6	56·9		24·8	57·3		24·9	57·6			
	2	33·4	73·5		33·9	74·7		34·4	75·9		34·8	76·9		35·1	77·8		35·4	78·5		35·7	79·2		36·0	79·9		36·2	80·5		36·4	81·1		36·6	81·6			
	4	48·5	104·0		49·3	105·9		50·0	107·4		50·6	108·8		51·1	110		51·6	111·1		52·0	112·1		52·4	113·1		52·8	113·9		53·1	114·6		53·4	115·3			
0·7	0·5	15·9	38·5	89·9	16·0	39·0	93·2	16·3	39·9	96·4	16·5	40·5	99·1	16·7	41·1	102	16·9	41·5	104	17·0	41·9	106	17·1	42·3	108	17·2	42·6	110	17·3	42·9	112	17·4	43·2	103			
	1	23·6	54·4		24·0	55·5		24·4	56·4		24·7	57·2		25·0	58·0		25·2	58·6		25·4	59·2		25·6	59·8		25·8	60·2		26·0	60·7		26·2	61·1			
	2	34·8	77·0		35·4	78·4		35·9	79·8		36·4	80·9		36·8	82·0		37·2	82·9		37·5	83·8		37·8	84·5		38·1	85·2		38·3	85·9		38·6	86·4			
	4	50·5	108·9		51·5	111·0		52·4	112·9		53·0	114·5		53·6	116·0		54·2	117·3		54·7	118·5		55·2	119·5		55·6	120·5		56·0	121·5		56·4	122·2			
0·8	0·5	16·4	40·0	96·7	16·7	40·8	101	16·9	41·5	105	17·1	42·2	108	17·3	42·8	111	17·5	43·3	114	17·7	43·8	116	17·8	44·2	118	17·9	44·6	121	18·0	45·0	123	18·1	45·3	124			
	1	24·2	56·6		24·8	57·8		25·3	58·8		25·6	59·8		25·9	60·6		26·2	61·3		26·4	62·0		26·7	62·6		26·9	63·2		27·1	63·7		27·2	64·2			
	2	36·1	80·0		36·7	81·7		37·2	83·2		37·7	84·4		38·2	85·6		38·6	86·7		39·0	87·6		39·4	88·5		39·7	89·3		40·0	90·4		40·3	90·7			
0·9	0·5	16·8	41·3	103	17·1	42·2	108	17·4	43·0	112	17·6	43·7	116	17·8	44·4	119	18·0	45·0	123	18·2	45·5	126	18·4	46·0	128	18·5	46·5	131	18·6	46·9	133	18·7	47·3	135			
	1	25·2	58·4		25·6	59·7		26·0	60·8		26·4	61·8		26·8	62·8		27·1	63·3		27·4	64·4		27·6	65·0		27·8	65·7		28·0	66·3		28·2	66·9			
	2	37·1	82·6		37·8	84·4		38·4	86·1		39·0	87·6		39·5	88·8		40·0	90·0		40·5	27·3		40·9	91·9		41·2	92·9		41·5	93·8		41·8	94·6			
1·0	0·5	17·2	42·4	109	17·5	43·4	114	17·8	44·3	119	18·1	45·1	123	18·3	45·8	127	18·5	46·5	131	18·7	47·1	134	18·9	47·6	137	19·0	48·1	140	19·1	48·5	143	19·2	49·0	145			
	1	25·7	60·0		26·2	61·4		26·7	62·7		27·1	63·8		27·5	64·8		27·8	65·7		28·1	66·6		28·4	67·3		28·7	68·0		28·9	68·6		29·0	69·3			
	2	38·0	84·8		38·7	86·8		39·4	88·6		40·0	90·2		40·6	91·6		41·2	92·9		41·6	94·1		42·0	95·2		42·4	96·2		42·8	97·1		43·0	98·0			
1·1	0·5	17·5	43·4	114	17·8	44·5	120	18·1	45·4	124	18·4	46·3	130	18·7	47·1	134	18·9	47·8	138	19·1	48·4	142	19·3	49·0	145	19·5	49·6	149	19·6	50·1	152	19·7	50·5	155			
	1	26·2	61·4		26·8	63·0		27·3	64·2		27·7	65·4		28·1	66·6		28·5	67·6		28·8	68·5		29·1	69·4		29·4	70·2		29·6	70·8		29·8	71·5			
	2	38·7	86·8		39·6	89·0		40·3	90·9		41·0	92·5		41·6	94·2		42·2	95·6		42·7	96·9		43·2	98·0		43·6	99·2		44·0	100·1		44·4	101·1			
1·2	0·5	17·8	44·3	119	18·1	45·4	125	18·5	46·5	131	18·8	47·4	136	19·0	48·2	141	19·2	49·0	145	19·5	49·7	149	19·7	50·3	153	19·9	50·9	157	20·1	49·7	160	20·2	52·0	163			
	1	26·7	62·7		27·3	64·2		27·8	65·7		28·3	67·0		28·6	68·2		29·0	69·3		29·3	70·3		29·7	71·2		30·0	71·9		30·3	72·8		30·5	73·5			
	2	38·4	88·6		40·3	90·9		41·2	92·9		41·9	94·8		42·5	96·4		43·0	98·0		43·6	99·3		44·1	100·7		44·6	101·9		45·0	103·0		45·4	104·0			
1·3	0·5	18·1	45·1	123	18·4	46·3	130	18·8	47·4	136	19·1	48·3	142	19·3	49·2	147	19·6	50·1	152	19·8	50·8	156	20·0	51·5	161	20·2	52·0	165	20·4	52·7	168	20·6	53·2	172			
	1	27·1	63·8		27·7	65·4		28·3	67·0		28·7	68·4		29·2	69·7		29·6	70·8		30·0	71·7		30·3	72·8		30·6	73·7		30·9	74·6		31·1	75·3			
	2	40·0	90·2		41·0	92·5		41·9	94·8		42·7	96·7		43·4	98·5		44·0	100·2		44·6	101·6		45·1	103·1		45·5	101·3		45·9	105·4		46·3	106·8			
1·4	0·5	18·3	45·8	127	18·7	47·1	134	19·0	48·2	141	19·3	49·2	147	19·6	50·2	152	19·9	51·0	158	20·2	51·8	163	20·4	52·6	167	20·6	53·2	171	20·8	53·9	176	20·9	54·4	179			
	1	27·5	64·8		28·1	66·6		28·6	68·2		29·2	69·7		29·6	71·0		30·1	72·2		30·4	73·3		30·8	74·35		31·1	75·1		31·4	76·2		31·7	77·0			
	2	40·6	91·6		41·6	94·2		42·5	96·4		43·4	98·5		44·0	100·4		44·7	102·1		45·3	103·6		45·9	105·2		46·4	106·5		46·9	107·9		47·4	108·9			
1·5	0·5	18·5	46·5	131	18·9	47·8	138	19·2	49·0	145	19·6	50·1	152	19·9	51·0	158	20·2	52·0	163	20·4	52·8	169	20·6	53·6	174	20·8	54·2	178	21·0	54·9	183	21·2	55·5	187			
	1	27·8	65·7		28·5	67·6		29·0	69·3		29·6	70·8		30·1	72·2		30·5	73·5		30·9	74·7		31·3	75·7		31·6	76·7		31·9	77·7		32·2	78·6			
	2	41·2	92·9		42·2	95·6		43·0	98·0		44·0	100·2		44·7	102·1		45·4	104·0		46·0	105·5		46·6	107·1		47·2	108·5		47·7	109·8		48·2	111·1			
1·6	0·5	18·7	47·1	134	19·1	48·4	142	19·5	49·7	149	19·8	50·8	156	20·2	51·8	163	20·4	52·8	168	20·6	53·6	174	20·9	54·5	179	21·2	55·2	185	21·4	56·0	189	21·5	56·6	194			
	1	28·1	66·6		28·8	68·5		29·3	70·3		30·0	71·7		30·4	73·3		30·9	74·7		31·4	75·9		31·7	77·0		32·1	78·1		32·4	79·0		32·7	80·0			
	2	41·6	94·1		42·7	96·9		43·6	99·3		44·6	101·6		45·3	103·6		46·0	105·5		46·6	107·4		47·3	108·9		47·9	110·5		48·4	112·0		48·9	113·2			
1·7	0·5	18·9	47·6	137	19·3	49·0	145	19·7	50·3	153	20·0	51·5	161	20·4	52·6	167	20·6	53·6	174	20·9	54·5	179	21·2	55·4	185	21·4	56·1	190	21·6	56·8	195	21·8	57·5	200			
	1	28·4	67·3		29·1	69·4		29·7	71·2		30·3	72·8		30·8	74·3		31·3	75·7		31·7	77·0		32·2	78·2		32·5	79·3		32·9	80·4		33·2	81·3			
	2	42·0	95·2		43·2	98·0		44·1	100·7		45·1	103·1		45·9	105·2		46·6	107·1		47·3	108·9		48·0	110·6		48·6	112·0		49·1	113·6		49·6	115·0			
1·8	0·5	19·0	48·1	140	19·5	49·6	149	19·9	50·9	157	20·2	52·0	164	20·6	53·2	172	20·8	54·2	178	21·2	55·2	185	21·4	56·1	190	21·6	56·9	196	21·8	57·7	201	22·0	58·4	206			
	1	28·7	68·0		29·4	70·2		30·0	71·9		30·6	73·7		31·1	75·1		31·6	76·7		32·1	78·1		32·5	79·3		32·9	80·5		33·2	81·6		33·6	82·6			
	2	42·4	96·2		43·6	99·2		44·6	101·9		45·5	104·3		46·4	106·5		47·2	108·5		47·9	110·5		48·6	112·0		49·3	113·7		49·9	115·7		50·4	116·8			
1·9	0·5	19·1	48·5	143	19·6	50·1	152	20·1	49·7	160	20·4	52·7	168	20·8	53·9	176	21·0	54·9	183	21·4	56·0	189	21·6	56·8	196	21·8	57·7	201	22·1	58·4	207	22·3	59·2	212			
	1	28·9	68·6		29·6	70·8		30·3	72·8		30·9	74·6		31·4	76·2		31·9	79·0		32·4	79·0		32·9	80·4		33·2	81·6		33·6	82·7		34·0	83·8			
	2	42·8	97·1		44·0	100·1		45·0	103·0		45·9	105·4		46·9	107·9		47·7	112·0		48·4	112·0		49·1	113·6		49·9	115·7		50·3	116·9		50·9	118·0			
2·0	0·5	19·2	49·0	145	19·7	50·5	155	20·2	52·0	163	20·6	53·2	172	20·9	54·4	180	21·2	55·5	187	21·5	56·6	194	21·8	57·5	200	22·0	58·4	206	22·3	59·2	212	22·5	60·0	218			
	1	29·0	69·3		29·8	71·5		30·5	73·5		31·1	75·3		31·7	77·0		32·2	78·6		32·7	80·0		33·2	81·3		33·6	82·6		34·0	83·8		34·4	84·8			
	2	43·0	98·0		44·4	101·1		45·5	104·0		46·3	104·0		47·4	108·9		48·2	111·1		48·9	113·2		49·6	115·0		50·4	116·8		50·9	118·0		51·5	120·0			

To face p. 56.]

The train consumption, making the 9 per cent. allowance for rotational momentum, is equal to—

$$(0{\cdot}0303 \times F^2) \text{ w hr per ton}$$

$$\text{or} \quad (218 \times C \times M) \text{ w hr per ton}$$

$$= \left(218 \times \frac{C \times M}{M}\right) \text{ w hr per ton-mile}$$

$$= (218 \times C) \text{ w hr per ton-mile.}$$

This result shows that the limiting train consumption per ton-mile for any given values of acceleration and deceleration is, under the assumed conditions, the same for any length of run. Q, the duration of stops, may usually be taken as 20 seconds. The highest attainable schedule speeds, and the corresponding crest values, have been calculated, by means of the above formulae, for runs of 0·5, 1, 2

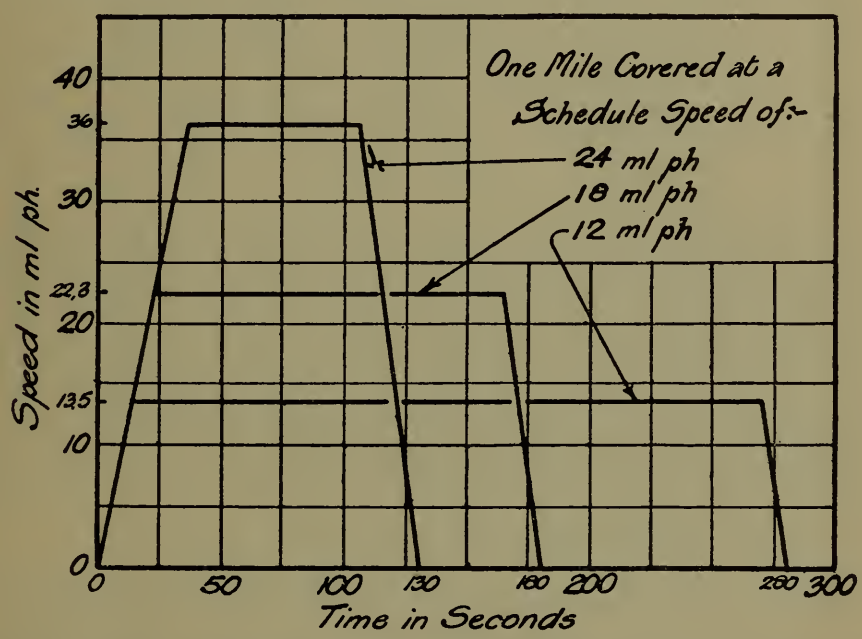


FIG. 31.—Speed-time Diagram for 1-mile Run covered at Schedule Speeds of 12, 18, and 24 ml ph, with Constant Acceleration and Braking, and neglecting Train-friction.

and 4 miles, and for various rates of acceleration and deceleration. The results are embodied in Table XIX. (folding table).

As an instructive instance of the application of these principles, let us consider the case of a 1-mile run from start to stop, *i.e.* let us consider a case where M = 1·0. Let us first study this run for schedule speeds of 12, 18, and 24 ml ph. The speed-time diagrams corresponding to these three schedule speeds, for A = 1·0 and B = 1·5, are readily worked out from the formulae and methods already described, and are plotted in Fig. 31.

Assuming 100 per cent. efficiency for the electrical equipment and negligible train-friction, the input required will be exclusively that corresponding in each case to the momentum at the crest speed, and may be obtained by the formula—

$$\text{Translational momentum} = 0{\cdot}0278 \times F^2$$
$$\text{Total momentum} = 0{\cdot}0278 \times 1{\cdot}09 \times F^2$$
$$= 0{\cdot}0303 \times F^2 \text{ w hr per ton}$$

where F is the crest speed in ml ph. The calculated values of the energy consumption for these three cases are shown in Table XX.—

TABLE XX.—ENERGY CONSUMPTION FOR 1-MILE RUN AT VARIOUS SCHEDULE SPEEDS, ASSUMING NO TRAIN-FRICTION AND 100 PER CENT. EFFICIENCY OF THE ELECTRICAL EQUIPMENT.

Schedule speed (ml ph).	F, the crest speed (ml ph).	Train consumption (w hr per ton-mile).
12	13·5	5·5
18	22·3	15·0
24	36	39·2

It is evident that, with increasing speed, we are rapidly approaching a schedule speed which cannot be exceeded with the assumed values of acceleration, deceleration, distance between stops and duration of stop. The limiting schedule speed is that at which the brakes must be applied at the very instant that the acceleration has been completed, *i.e.* there is in the corresponding speed-time diagram no interval of running at constant speed. For the assumed acceleration and deceleration, and for 20-second stops, this speed may be calculated from the formula already given on p. 56, namely—

$$\text{Limiting schedule speed} = \frac{3600 \times C \times M}{\sqrt{7200 \times C \times M} + Q \times C}$$

For our case M = 1 and Q = 20. C may be obtained from Table XV.

In this case, since A = 1·0 and B = 1·5—

$$C = 0{\cdot}60$$

$$\therefore \text{Limiting schedule speed} = \frac{2160}{\sqrt{4320} + 12}$$
$$= 27{\cdot}8 \text{ ml ph.}$$

Consequently, the time required for the 1-mile run from start to stop is—

$$\frac{3600}{27{\cdot}8} - 20 = 109{\cdot}6 \text{ seconds.}$$

The average speed is—

$$\frac{109{\cdot}6 + 20}{109{\cdot}6} \times 27{\cdot}8 = 32{\cdot}8 \text{ ml ph.}$$

For this case, the speed-time diagram for which is drawn in Fig. 32, F, the crest value of the speed, is obviously equal to twice the average speed. Thus—

$$F = 2 \times 32{\cdot}8 = 65{\cdot}6 \text{ ml ph.}$$

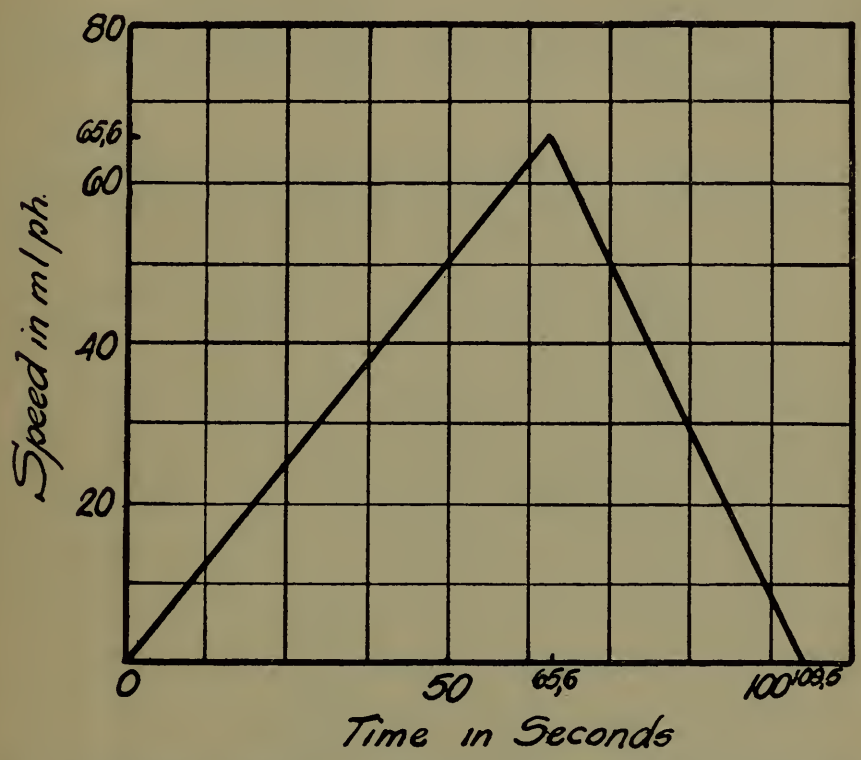


FIG. 32.—Speed-time Diagram for 1-mile Run under Limiting Conditions. No Coasting Period; Acceleration and Deceleration of 1·0 and 1·5 ml phps respectively.

We could have obtained F directly from the formula already given on p. 56, namely—

$$F = \sqrt{7200 \times C \times M}$$

For $M = 1{\cdot}00$

$C = 0{\cdot}60$

$$\therefore F = \sqrt{4320} = 65{\cdot}6 \text{ ml ph.}$$

In order to obtain higher schedule speeds we must, obviously, resort to higher accelerating or decelerating rates, or both. Let us work out the case of a schedule speed of 30 ml ph, using an accelerating rate of 1·3 ml phps and a decelerating rate of 1·8 ml phps. The calculation may be arranged in an orderly form as follows :—

Schedule speed	30 ml ph.
Length of run from start to stop .	1 mile.
Number of stops per hour . . .	30.
Time at rest per hour	$30 \times 20 = 600$ seconds.
Running time per hour	$3600 - 600 = 3000$ seconds.
Time occupied by the 1-mile run from start to stop	$\frac{3000}{30} = 100$ seconds.
Accelerating rate	1·3 ml phps.
Decelerating rate	1·8 ml phps.

Let the crest speed be F ml ph. The values in Table XXI. are directly deduced.

TABLE XXI.—SHOWING THE DURATION OF, AND THE AVERAGE SPEED AND DISTANCE COVERED DURING, THE ACCELERATING, CONSTANT SPEED, AND DECELERATING PERIODS OF THE SPEED-TIME DIAGRAM *f* IN FIG. 33.

—	Duration of run (seconds).	Duration (hours).	Average speed (ml ph).	Distance covered (mile).
Accelerating period	$\frac{F}{1{\cdot}3}$	$\frac{1}{3600} \times \frac{F}{1{\cdot}3}$	$\frac{F}{2}$	$\frac{1}{3600} \times \frac{F}{1{\cdot}3} \times \frac{F}{2}$
Constant speed period	$100 - \frac{F}{1{\cdot}3} - \frac{F}{1{\cdot}8}$	$\frac{1}{3600}\left(100 - \frac{F}{1{\cdot}3} - \frac{F}{1{\cdot}8}\right)$	F	$\frac{1}{3600}\left(100 - \frac{F}{1{\cdot}3} - \frac{F}{1{\cdot}8}\right)F$
Decelerating period	$\frac{F}{1{\cdot}8}$	$\frac{1}{3600} \times \frac{F}{1{\cdot}8}$	$\frac{F}{2}$	$\frac{1}{3600} \times \frac{F}{1{\cdot}8} \times \frac{F}{2}$

The total distance covered during the run is one mile—

$$\therefore \frac{1}{3600}\left(\frac{F^2}{2{\cdot}6} + 100F - \frac{F^2}{1{\cdot}3} - \frac{F^2}{1{\cdot}8} + \frac{F^2}{3{\cdot}6}\right) = 1$$

$$0{\cdot}663F^2 - 100F = -3600$$

$$F^2 - 151F + 5700 = -5430 + 5700$$

$$F - 75{\cdot}5 = \pm\sqrt{270} = \pm 16{\cdot}4$$

$$F = 59{\cdot}1$$

Crest speed	= 59·1 ml ph
Time of accelerating period	= 45·5 seconds
„ constant speed „	= 21·6 „
„ decelerating „	= 32·9 „

The results of all these five cases, together with results for the train consumption, in w hr per ton-mile (estimated from the formula, Momentum $= 0{\cdot}0278 \times 1{\cdot}09 \times F^2$), are given in Table XXII., also values for a schedule speed of 27 ml ph.

TABLE XXII.—ENERGY CONSUMPTION OF TRAINS STOPPING ONCE PER MILE AND RUNNING AT DIFFERENT SCHEDULE SPEEDS, ASSUMING NO TRAIN-FRICTION AND 100 PER CENT. EFFICIENCY.

Schedule speed (ml ph).	Acceleration (ml phps).	Deceleration (ml phps).	Crest speed (ml ph).	Ratio of crest to schedule speed.	Energy required (w hr per ton-mile).
12	1·0	1·5	13·5	1·125	5·5
18	1·0	1·5	22·3	1·24	15·1
24	1·0	1·5	36·0	1·5	39·2
27·8	1·0	1·5	65·6	2·37	131
27	1·3	1·8	42·3	1·57	54
30	1·3	1·8	59·1	1·97	106

The corresponding speed-time diagrams are brought together in Fig. 33.

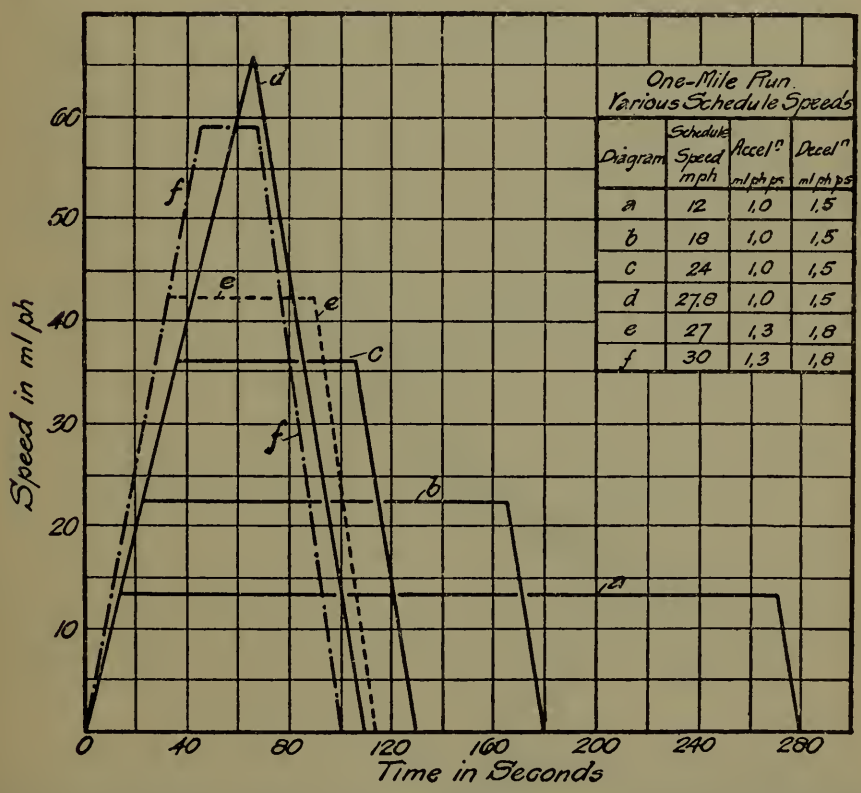


FIG. 33.—Speed-time Diagrams for 1-mile Run covered at the Various Schedules indicated. No Train-friction.

In Table XXII. two different combinations of accelerating and decelerating rates are considered, but calculations have been made for other cases, and the results are given in Fig. 35, in which the values

of the amounts of energy required in w hr per ton-mile for these 1-mile runs are plotted as ordinates, with the schedule speeds in ml ph as abscissae. The full lines represent the results for the separate values

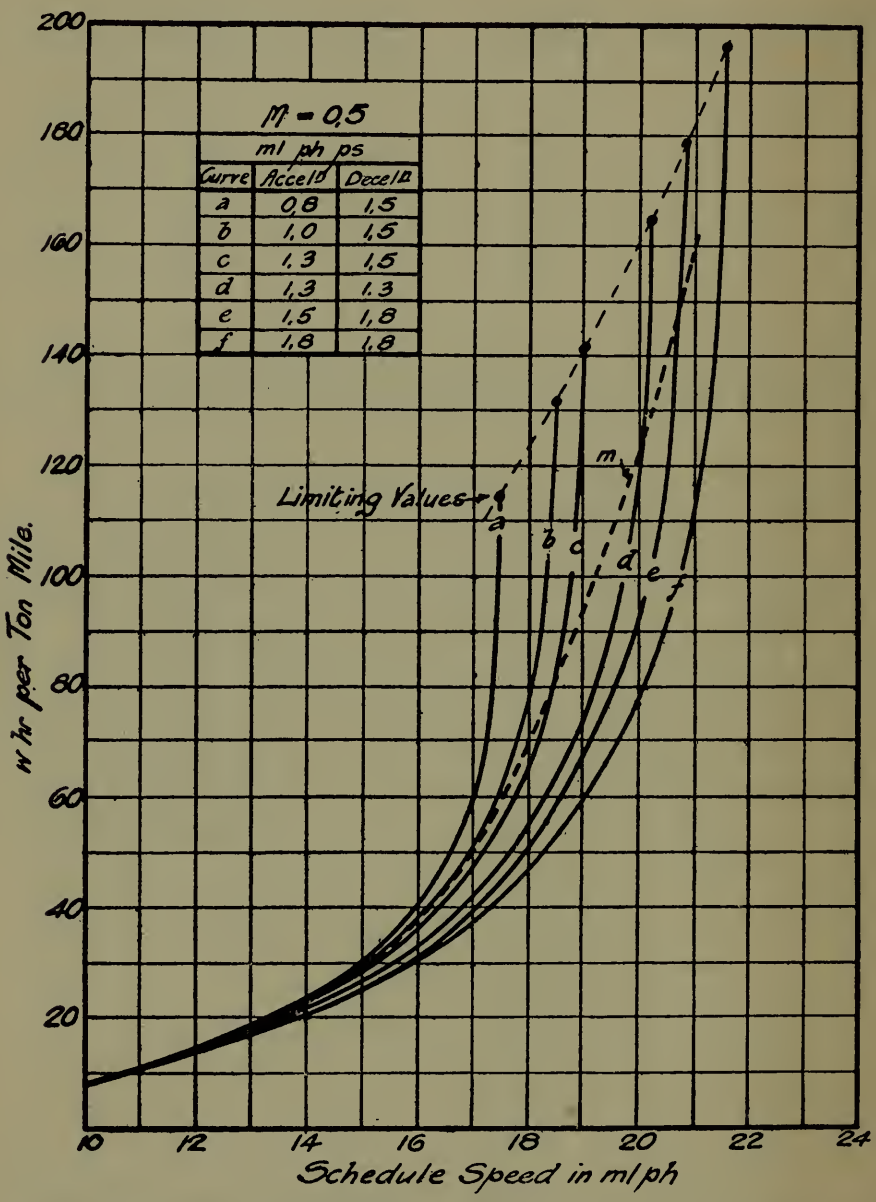


FIG. 34.—Curves of Train Consumption in w hr per ton-mile, assuming no Train-friction and 100 per cent. Efficiency of Equipment, for 0·5-mile Run and Various Schedule Speeds, with the Different Accelerations and Decelerations shown.

of acceleration and deceleration, while the dotted line *m* is a mean curve drawn for a purpose, the utility of which will be obvious from the following considerations.

For definite accelerations and decelerations we obtain corresponding curves for the energy required, with limiting values for each, corresponding to the cases where the constant-speed interval disappears. These limiting values are seen to lie on an even curve.

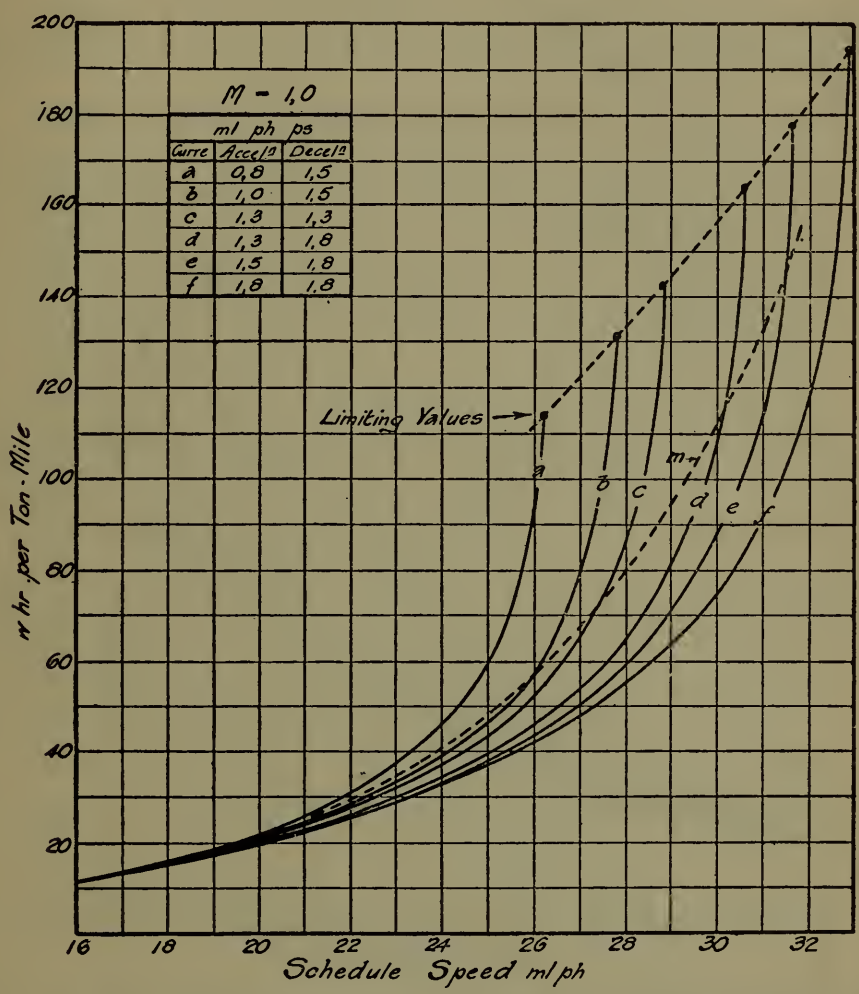


FIG. 35.—Curves of Train Consumption in w hr per ton-mile, assuming no Train-friction and 100 per cent. Efficiency of Equipment, for 1-mile Run and Various Schedule Speeds, with the Different Accelerations and Decelerations shown.

In Figs. 34 and 36 are plotted, for 0·5 and 2-mile runs, similar series of these curves for the same respective accelerations and decelerations indicated. The limiting values are again clearly shown at the top of the curves.

Strict comparisons of the energy consumption for any given length of run, at the required schedule speed, should be made for various accelerations and decelerations, since Figs. 34, 35, and 36

indicate the important influence which these rates have on the consumption.

For economic working, a train would not be run at the extreme limiting values shown in the curves. Nevertheless, the accelerations would be kept moderate, and the dotted curves *m* in Figs. 34, 35, and 36 are intended, in each case, to afford a locus for rational values for the range considered.

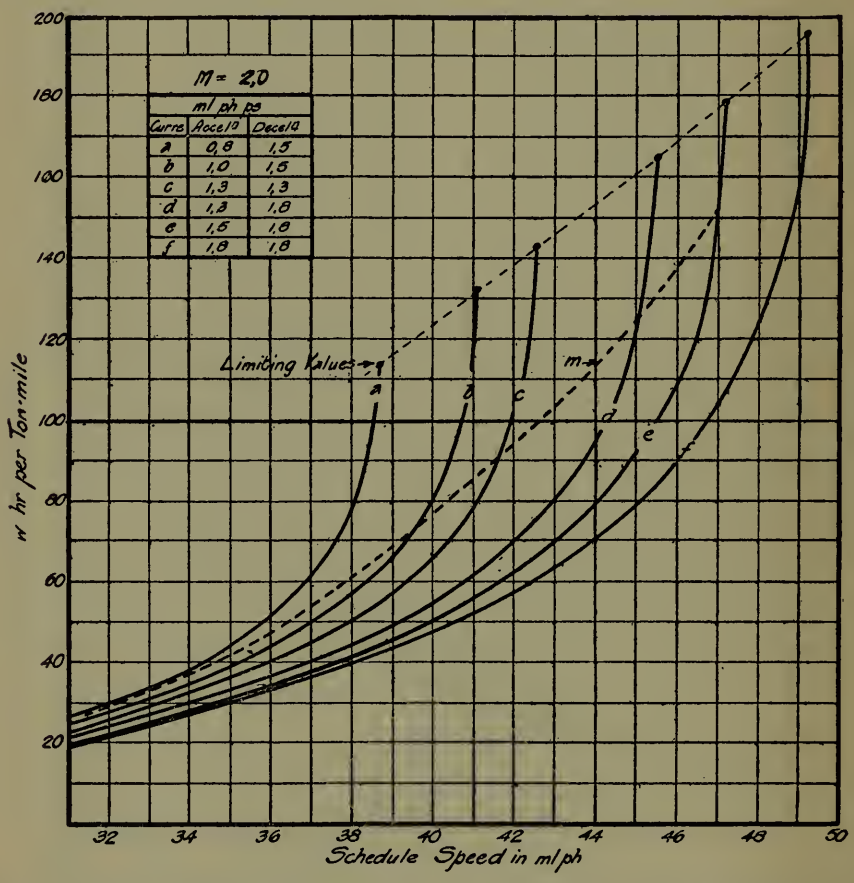


FIG. 36.—Curves of Train Consumption in w hr per ton-mile, assuming no Train-friction and 100 per cent. Efficiency of Equipment, for 2-mile Run and Various Schedule Speeds, with the Different Accelerations and Decelerations shown.

These curves *m*, for 0·5-, 1-, and 2-mile runs, are brought together in Fig. 38, and show rational values for the energy consumptions in w hr per ton-mile. In Fig. 37 the corresponding values for the w hr per ton are plotted.

Looking back to Fig. 34, relating to runs over distances of 0·5 mile from start to stop, we see that, neglecting train-friction and assuming 100 per cent. efficiency for the electrical equipment, the energy consumption for a schedule speed of—for instance—18 ml ph, may be anywhere from 50 w hr per ton-mile up to 100 w hr per ton-mile,

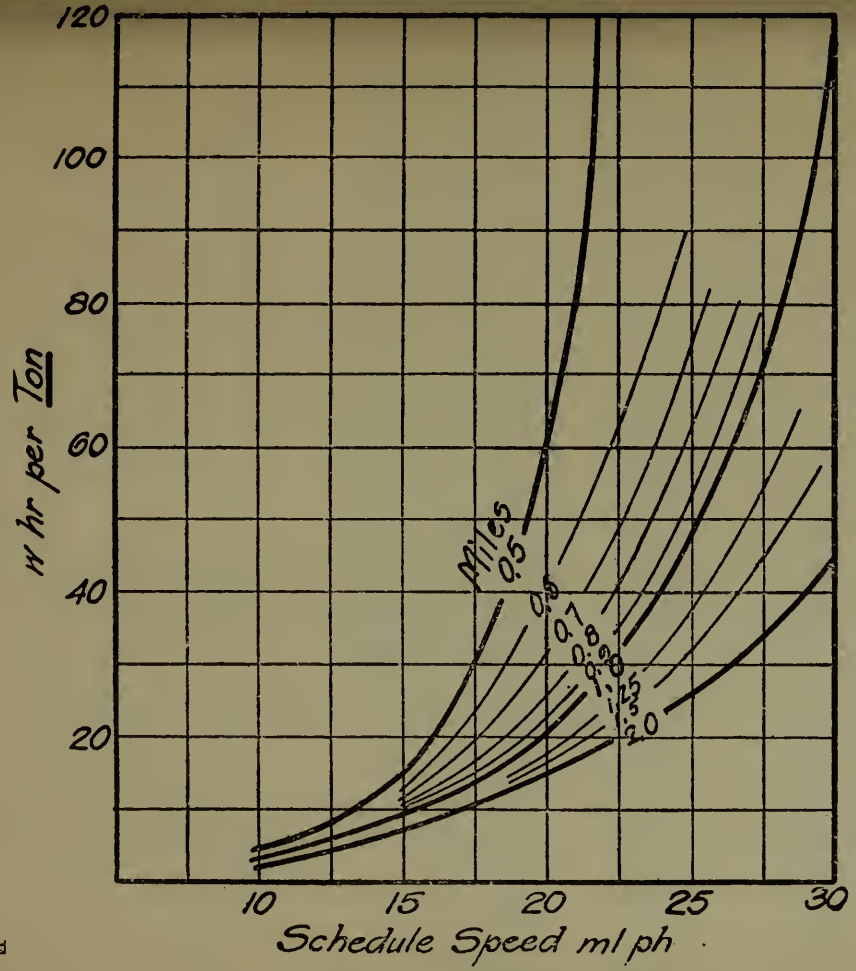


FIG. 37.—Curves of Train Consumption from Start to Stop, in w hr per *Ton* for Various Schedules. These are Mean Curves derived from Figs. 34, 35, and 36, and therefore involve the Assumption of no Train-friction and of 100 per cent. Efficiency of the Electrical Equipment.

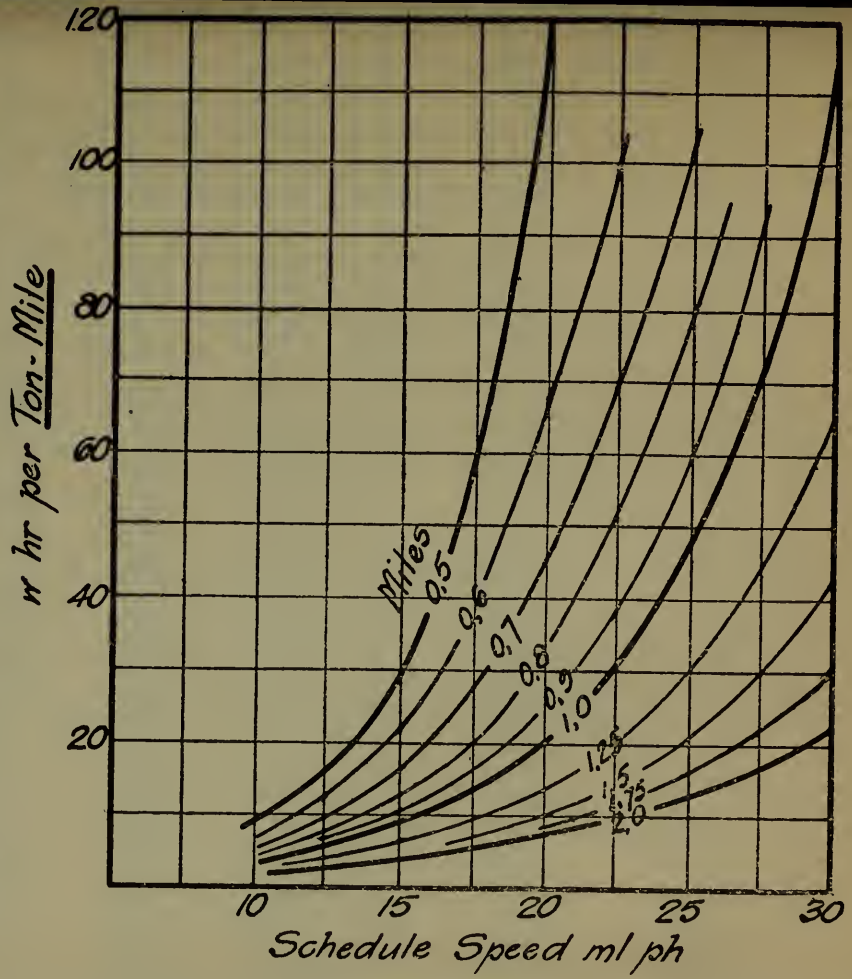


FIG. 38.—Curves of Train Consumption in w hr per *Ton-mile* for Various Schedules. These are Mean Curves derived from Figs. 34, 35, and 36, and therefore involve the Assumption of no Train-friction and of 100 per cent. Efficiency of the Electrical Equipment.

according as the mean of the acceleration and deceleration is nearer 1·8 or 1·2 ml phps.

Questions.—Chapter IV.

1. Assuming a speed-time diagram as in Fig. 18, (*a*) What is the crest speed necessary to traverse a distance of 0·75 mile at an average speed of 20 ml ph, when the acceleration and deceleration are 0·8 and 1·7 ml phps respectively? (*b*) Find the energy input for momentum.

Ans. (*a*) 23·9 ml ph.
(*b*) 17·3 w hr per ton.

2. A distance of one mile has to be covered at 18 ml ph : A = 0·9, B = 1·8. (*a*) First running continuously, (*b*) then with one stop halfway, and (*c*) finally with two stops at one-third and two-thirds of the distance.

The stops may be taken as at signals, but to be only of instantaneous duration. Determine the necessary crest speeds in these cases, using the formula of p. 48.

Ans. (*a*) 20 ml ph.
(*b*) 22 ml ph.
(*c*) 27·4 ml ph.

3. Repeat question 2, but take the stops as of 3 seconds' duration at each signal, and ascertain the crest speeds then necessary.

Ans. (*a*) 20 ml ph.
(*b*) 23·3 ml ph.
(*c*) 33·5 ml ph.

4. Following on the idea of question 2: (*a*) Find the value of C which permits of five stops, one every one-fifth mile, for an instant, but under limiting conditions (as in Fig. 32). The *crest* speed will, of course, be 36 ml ph. (*b*) Given A = B, what will be the values of A and B?

Ans. (*a*) C = 0·9.
(*b*) A = B = 1·8.

5. Determine (*a*) the crest speed for the limiting run (similar to Fig. 32) over 1000 m, when the values for acceleration and braking are respectively 0·8 and 1·5 ml phps; and (*b*) the schedule speed when the stop is of 20 seconds' duration.

Ans. (*a*) 48·3 ml ph.
(*b*) 20·0 ml ph.

6. Under the assumed frictionless conditions, a train runs 1·20 mile at a schedule speed of 25 ml ph, and with a 20-second stop. Compare the energy consumptions, (*a*) when A = 1·0, B = 1·5; and (*b*) with the limiting speed-time diagram and A = B.

Ans. (*a*) 36·6 w hr per ton for each run from start to stop.
30·5 w hr per ton-mile.
(*b*) 97·0 w hr per ton for each run from start to stop.
80·8 w hr per ton-mile.

CHAPTER V

THE EFFICIENCY OF THE ELECTRICAL EQUIPMENT

In Fig. 39 is given the experimentally observed speed-time diagram of a train weighing 72 tons, and equipped with four 150-hp motors. The train comprised one motor-coach and one trailer, and the speed-time diagram relates to a run of 1 mile from start to stop. In the tests, which were made by the engineers of the Lancashire and Yorkshire Railway * on the electrified section between Liverpool and Southport, the 1-mile run was accomplished in 120 seconds. The *average* speed was thus—

$$\frac{3600}{120} \times 1 = 30 \text{ ml ph.}$$

This, with 20-second stops, corresponds to a *schedule* speed of—

$$\frac{120}{120 + 20} \times 30 = 25{\cdot}7 \text{ ml ph.}$$

The input, measured at the train, was 77·7 w hr per ton-mile.

From the speed-time diagram (Fig. 39) it is seen that the crest speed was 40 ml ph. Consequently, to supply momentum, there was required—

$$0{\cdot}0303 \times 40^2 = 48{\cdot}5 \text{ w hr per ton.}$$

The distance-time curve is shown in Fig. 40. Fifty-six seconds elapsed from the start up to the time when the electricity was cut off, and at the end of the 56th second the train had travelled 0·43 mile. Taking the train-friction as 6 kg per ton, then the portion of the input which is accounted for by train-friction was—

$$\frac{0{\cdot}43 \times 1609 \times 6}{367} = 11{\cdot}3 \text{ w hr per ton.}$$

The losses in the electrical equipment were consequently—

$$77{\cdot}7 - (48{\cdot}5 + 11{\cdot}3) = 17{\cdot}9 \text{ w hr per ton weight of train.}$$

The output from the electrical equipment amounted to—

$$48{\cdot}5 + 11{\cdot}3 = 59{\cdot}8 \text{ w hr per ton,}$$

* "Proceedings of the Institution of Civil Engineers," vol. clxxix. pt. 1, p. 134.

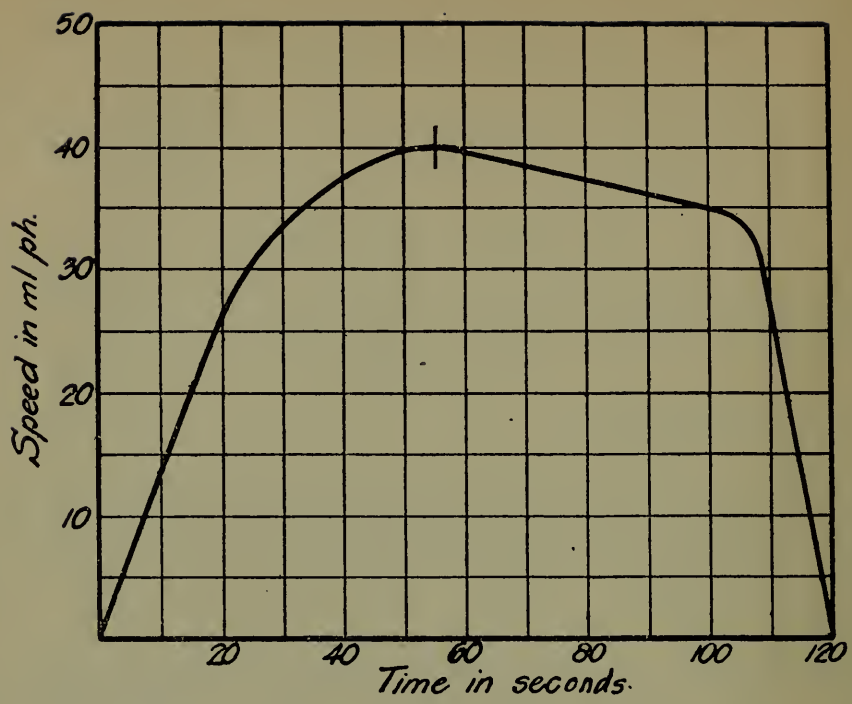


FIG. 39.—Lancashire and Yorkshire Railway Speed-time Diagram for 1·0-mile Run at Schedule Speed of 25·7 ml ph.

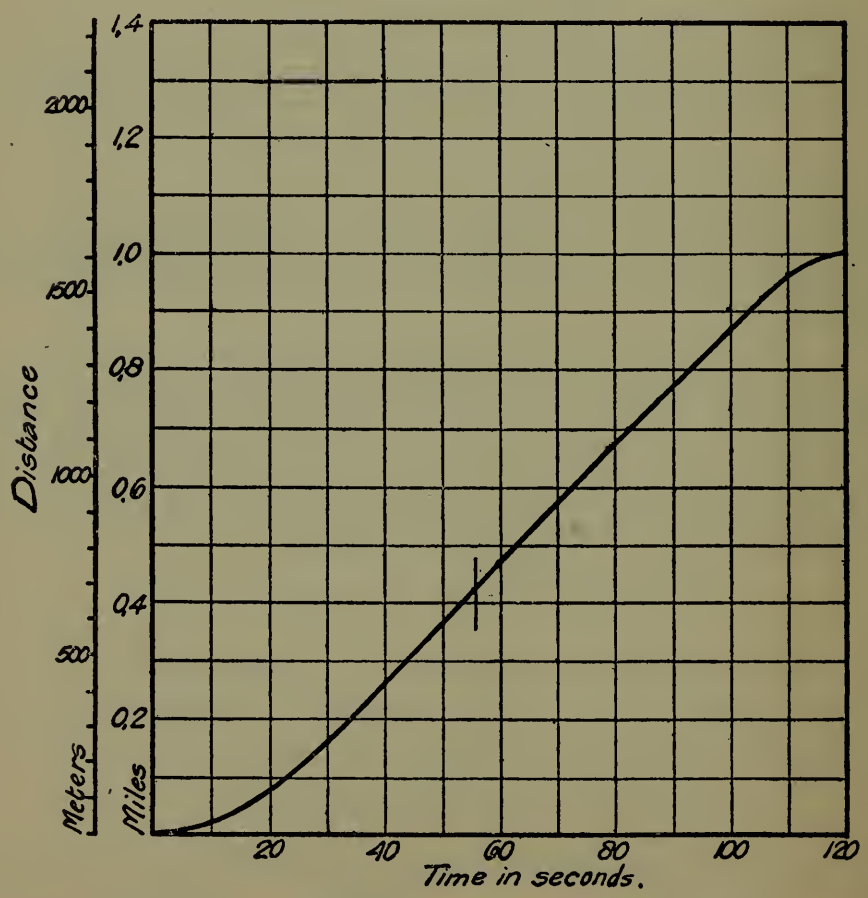


FIG. 40.—Distance-time Curve corresponding to Fig. 39.

and, since the input was 77·7 w hr per ton, the efficiency of the electrical equipment, under the conditions of this particular run, was—

$$\frac{59{\cdot}8}{77{\cdot}7} \times 100 = 76{\cdot}9 \text{ per cent.}$$

This is rather a high efficiency and is not often attained.

On p. 432 of Mr. Aspinall's Presidential Address, delivered before the Institution of Mechanical Engineers on April 23, 1909, another test made on the Lancashire and Yorkshire Railway is quoted. This test was made on a 3-coach train weighing 117 tons, and operated to a *schedule* speed of 30 ml ph, with 1 stop every 1·32 mile. The speed-time diagram was not given, but it is reasonable to conclude that it is fairly represented by Fig. 41. The corresponding distance-time curve is given in Fig. 42. The time occupied by a single run from start to stop is seen to have been 138 seconds. Consequently, the *average* speed was—

$$\frac{138 + 20}{138} \times 30 = 34{\cdot}3 \text{ ml ph.}$$

The crest value of the speed is seen, from Fig. 41, to have been 49 ml ph. The time elapsing from start to cut-off is 82 seconds, and the distance covered during this time is 0·75 mile, or 1200 m.

The train, which, as already stated, weighed 117 tons, was composed of three coaches, two being motor-coaches and the third a trailer. Each motor-coach carried four 150-hp motors. Consequently, the complete electrical equipment comprised eight 150-hp motors and the auxiliary apparatus required for their operation. The input to the train worked out at 96 w hr per ton-mile.

This input is made up of three components—

I. Energy required for supplying momentum. This is equal to—

$$0{\cdot}0303 \times 49^2 = 72{\cdot}7 \text{ w hr per ton,}$$

and

$$\left(\frac{72{\cdot}7}{1{\cdot}32} = \right)55{\cdot}0 \text{ w hr per ton-mile.}$$

II. Energy required to overcome train-friction up to the point of cut-off. The electricity is cut off 82 seconds from the instant of starting, when the train has covered a distance of 1200 m (0·75 mile).

The friction component is (assuming 6 kg per ton) consequently equal to—

$$\frac{6 \times 1200}{367} = 19{\cdot}6 \text{ w hr per ton,}$$

and

$$\left(\frac{19{\cdot}6}{1{\cdot}32} = \right)14{\cdot}8 \text{ w hr per ton-mile.}$$

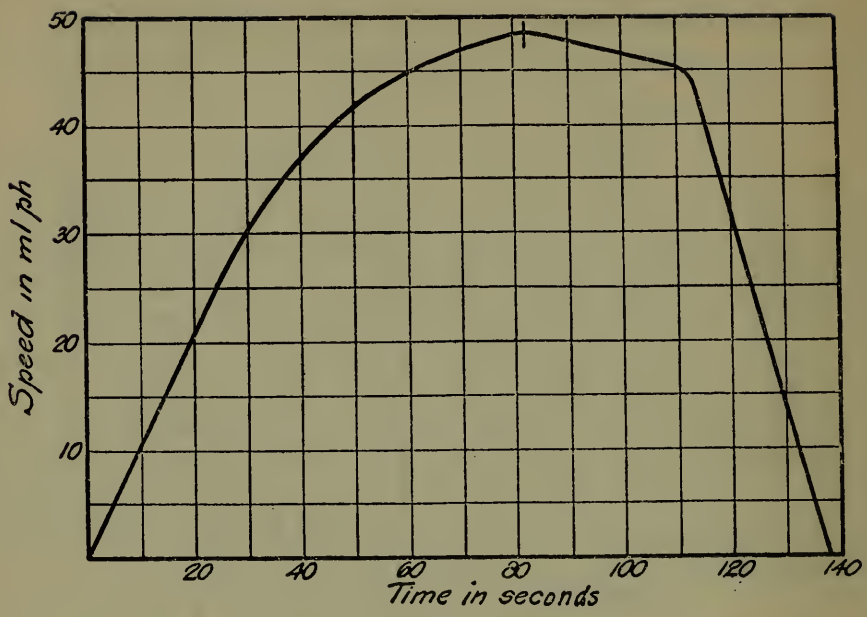


FIG. 41.—Lancashire and Yorkshire Railway Representative Speed-time Diagram for 1·32-mile Run at Schedule Speed of 30 ml ph.

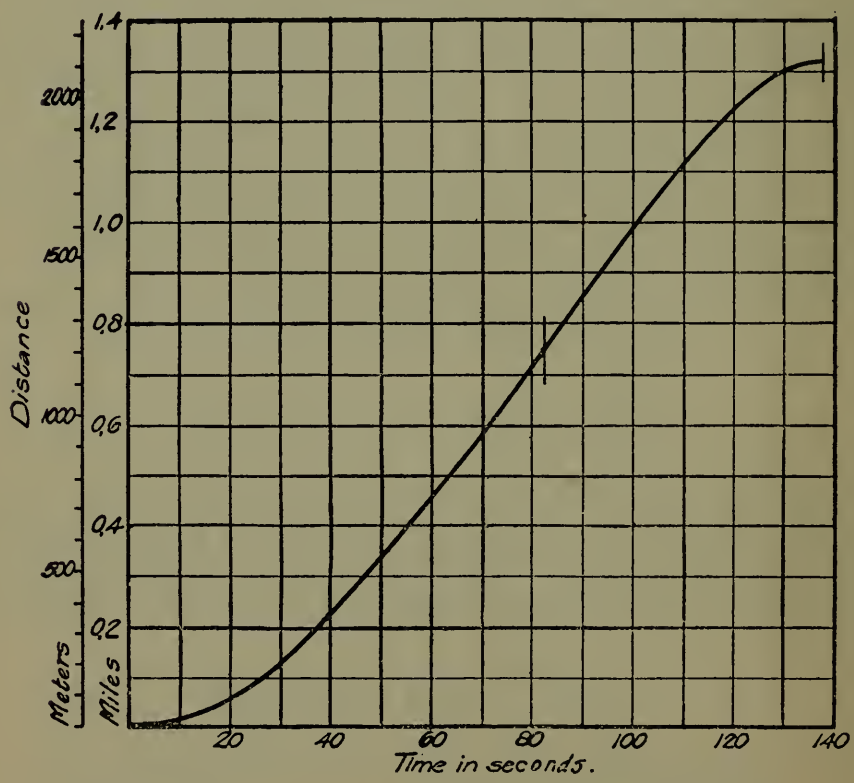


FIG. 42.—Distance-time Curve corresponding to Fig. 41.

III. Energy transformed from electricity into heat in the electrical equipment. This equals $96{\cdot}0 - (55{\cdot}0 + 14{\cdot}8) = 26{\cdot}2$ w hr per ton-mile.

The output from the electrical equipment is the sum of the momentum at the crest speed and of the train-friction up to the instant of cut-off, and is equal to—

$$55{\cdot}0 + 14{\cdot}8 = 69{\cdot}8 \text{ w hr per ton-mile.}$$

Since the input is 96·0 w hr per ton-mile, the efficiency is equal to—

$$\frac{69{\cdot}8}{96{\cdot}0} \times 100 = 72{\cdot}6 \text{ per cent.}$$

Let us further examine these two sets of results, *i.e.* the results of the tests on the 72-ton and 117-ton Lancashire and Yorkshire trains. The leading data is brought together in Table XXIII.—

TABLE XXIII.—ANALYSIS OF TRAIN TESTS ON THE LANCASHIRE AND YORKSHIRE RAILWAY.

Weight of train (ton)	72	117
Distance between stops (mile)	1·00	1·32
Schedule speed, assuming 20-second stops (ml ph)	25·7	30·0
Crest speed (ml ph)	40	49
Energy required for momentum (w hr per ton-mile)	48·5	55·0
Number of seconds elapsing up to cut-off	56	82
Distance up to point of cut-off (mile)	0·43	0·75
Energy required for train-friction up to point of cut-off (w hr per ton-mile)	11·3	14·8
Energy required to supply the losses in the electrical equipment (w hr per ton-mile)	17·9	26·2
Input to train (w hr per ton-mile)	77·7	96·0
Efficiency (per cent.)	76·9	72·6

The next steps of interest relate to the average input to the motors in the two cases. These are set forth in Table XXIV.—

TABLE XXIV.—CONTINUED ANALYSIS OF TRAIN TESTS ON THE LANCASHIRE AND YORKSHIRE RAILWAY.

Weight of train (ton)	72	117
Input per train-mile (kw hr)	5·60	11·2
Ditto in kw hr per hour, *i.e.* average kw input	144	336
Efficiency (from Table XXIII.)	0·769	0·726
Average output (kw)	111	244
Number of motors per train	4	8
Average output per motor, taken over entire run (hp), m	37·2	40·9
Ditto in per cent. of rated load	24·8	27·2
Number of seconds elapsing from start to "cut-off" of electricity	56	82
Number of seconds elapsing from start from one station to start from next station	140	158
Ratio of time during which train is consuming electricity to entire time of run, n	0·400	0·519
Average output per motor, during the time it is in circuit (hp) $m \div n$	93	79
Ditto in per cent. of rated load	62·0	52·6

The lower point of the efficiency curve at which the motors are working, as the result of the lower percentage of the rated load which they are carrying while in circuit, suffices to account for about half of the 4 per cent. lower over-all efficiency of the electrical equipment observed in the case of the 117-ton train. That this is the case will become clear by an examination of the efficiency curves of typical 150-hp series-wound railway motors. The curves, which correspond respectively to the operation of the motor at 250 volts (corresponding to the full series position of the controller) and at 500 volts (corresponding to the full parallel position of the con-

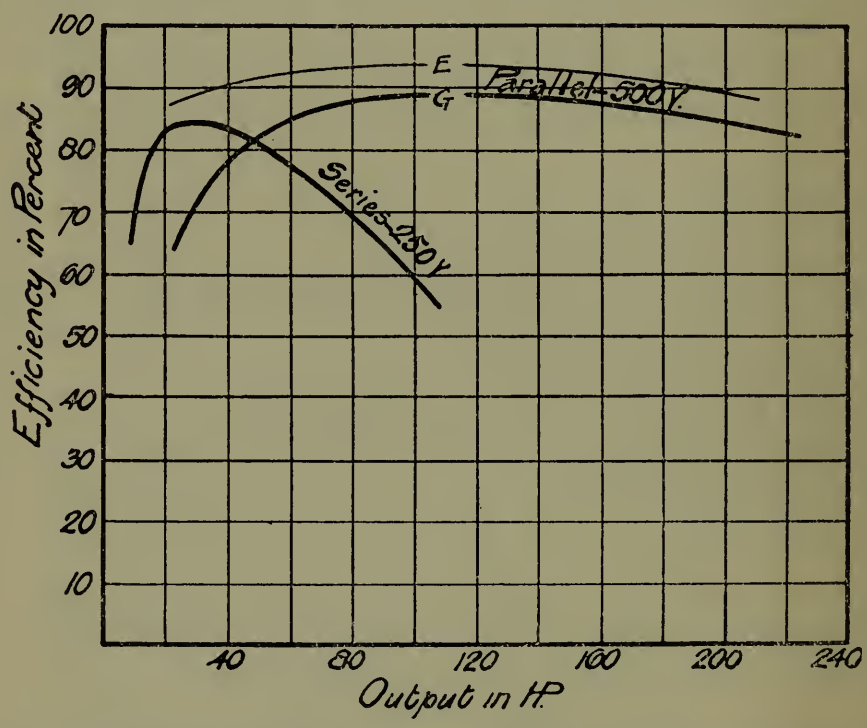


FIG. 43.—Representative Efficiency Curves for a 150-hp Continuous Electricity Railway Motor on 500 volts (Parallel) and 250 volts (Series).
Curve *E*, Excluding Gear.
,, *G*, Including Gear.

troller), are shown in Fig. 43. The results for 62 per cent. and 52·6 per cent. of rated load (from Table XXIV.) are as follows :—

	Load on motor.	
	62 per cent. of rated load (93 hp).	52·6 per cent. of rated load (79 hp).
Efficiency at 250 volts .	63·0 per cent.	70·0 per cent.
,, ,, 500 ,, .	89·0 ,,	87·5 ,,

The motors were in the series position for only some 10 to 15 seconds from the moment of starting, and during this time they were in series with resistances up to the very last few seconds. On the other hand, the motors were in parallel for some 45 to 65 seconds, and for most of this time they were running without resistances in series, *i.e.* they were running with about 500 volts at their terminals. Consequently, the average efficiencies of the motors for the entire run were far more influenced by the efficiencies in the parallel position, *i.e.* the 500-volt efficiencies, than by the 250-volt efficiencies. Taking into account the widely varying loads to which motors are subjected from instant to instant in a service of this kind, it appears sound to assess at 2 per cent. the amount by which the average motor *efficiency* for the entire run is less at 52·6 per cent. of rated load than at 62 per cent. of rated load.

Of the 4 per cent. lower over-all efficiency of the equipment, there is, in addition to the 2 per cent. just estimated, a further 2 per cent., which is accounted for as follows.

It will be observed that, during the first 20 seconds from the start, the 72-ton train was brought to a speed of 26 ml ph as against a speed of only 21 ml ph for the 117-ton train. The average accelerations for the two cases for the first 20 seconds were, consequently, 1·30 and 1·05 ml phps respectively. Now, the more rapid the acceleration the more quickly will the regulating rheostats, employed in series with the motors during starting, be cut out, and in the case of the test of the 72-ton train, the rheostats are known to have been finally cut out in the parallel operation only 15 seconds from the instant of starting the train, whereas, for the 117-ton train, the rheostats were probably in circuit (although this is only to be regarded as in the light of a reasonable estimate) for at least 30 seconds from starting the train. Consequently, losses were occurring in the controlling rheostats for only 10·7 per cent. of the entire time in the case of the 72-ton train, as against some 19 per cent. of the entire time in the case of the 117-ton train. In the latter case the rheostatic loss would have been sufficiently greater than in the former case to amply account for the remaining 2 per cent. deficit in the efficiency.

The points involved in the preceding paragraphs are of great importance. In the test of the 117-ton train, in spite of the severity of the schedule, the acceleration and the deceleration during braking have been kept within the limits of the best practice to which conformance can be made in routine daily service. In the test of the 72-ton train, however, the acceleration and the deceleration are well up to, and in fact are decidedly beyond, the all-round desirable values. Thus, the rate of deceleration during the last ten seconds of the run of the 72-ton train is seen from Fig. 39 to be of the very high value of 2·7 ml phps, while in the case of the 117-ton train, the deceleration

during the last ten seconds (Fig. 41) is at an approved rate, being 1·8 ml phps.

In the test with the 72-ton train the object has been to keep the crest speed down to a low value, with a view to obtaining a high efficiency. This has been accomplished by resorting to very high acceleration and deceleration. On the whole, the 76·9 per cent. efficiency obtained for the test run with the 72-ton train must be regarded as abnormal, since resort was made to exceedingly high acceleration and deceleration. The average outputs, *taken over the entire run*, and allowing for 20-second stops, are 37·2 hp for the 72-ton train and 40·9 hp for the 117-ton train. In terms of the rated load, these values work out at 24·8 per cent. and 27·2 per cent. respectively.

The question of motor heating is taken up again in Chapter XII., but let us at this point accept provisionally, for our present purposes, the rough rule that for a given service where the only intervals of rest are the 20-second stops at stations, and where each motor-coach must operate to its schedule for 18 consecutive hours, an aggregate rated capacity of electrical equipment equal to four times the average load must be provided. On the basis of the conventional method of rating railway motors, this will give reasonable assurance that the maximum temperature-rise above the temperature of the surrounding air will, in the conditions of actual service on the train, not appreciably exceed 65° C.

In the conventional basis of rating of railway motors it is required that after a one-hour test on a stand at the manufacturer's works at rated load (and with the covers over the commutator open), the temperature-rise of the hottest accessible part, as thermometrically determined, shall not exceed by more than 75° C., the temperature of the surrounding atmosphere. This is a purely nominal basis of rating; nevertheless, it has been proved, in the course of the many years during which it has been employed by all the leading manufacturers of railway motors, that it constitutes an excellent basis on which to rate motors. This 1-hour, 75° C. load, while it is of the order of four times the *average* load for which the motor is thermally suitable for a service of several consecutive hours, is a load well within the motor's capacity as regards mechanical construction and commutation. For brief periods during starting, a motor usually has to carry a load well up toward, or even above, its 1-hour, 75° C. rating; consequently, it is important that at its rated load the performance of the motor shall be excellent as regards commutation.

Corresponding to the efficiency investigations set out above for the Lancashire and Yorkshire equipments under the conditions of service, I have carried out a number of investigations of the working

efficiencies of the train equipments on several other electric railways.* The results indicate that for services where the trains operate at high schedule speeds with frequent stops, the over-all efficiency of the electrical equipment on the train should, in actual practice and for correctly driven, correctly proportioned trains and equipments, be of the order of 70 per cent. While it is often quite exact enough to employ this figure in estimates, nevertheless, I have attempted to assess more precise values for particular services, since the over-all efficiency is, in practice, a function of the length of run between stops, the schedule speed, and the acceleration and deceleration. The values at which I have arrived are set forth in Table XXV., and may

TABLE XXV.—OVER-ALL EFFICIENCIES OF ELECTRICAL EQUIPMENT FOR VARIOUS SCHEDULE SPEEDS AND RUNS.

Schedule speed from start to stop (ml ph).	Mean of Acceleration and Deceleration.											
	1·0 ml phps.				1·5 ml phps.				1·8 ml phps.			
	Distance between stops (miles).				Distance between stops (miles).				Distance between stops (miles).			
	0·5	1·0	2·0	4·0	0·5	1·0	2·0	4·0	0·5	1·0	2·0	4·0
10	76	—	—	—	74	76	—	—	74	76	—	—
15	71	75	78	—	72	75	77	—	72	75	77	—
20	68	72	76	78	70	73	76	79	70	73	76	79
25	65	70	74	77	67	71	75	78	67	71	75	78
30	—	68	72	76	65	69	73	77	—	70	74	77
35	—	65	71	74	—	67	72	75	—	68	73	76
40	—	—	69	73	—	66	71	74	—	67	72	75
45	—	—	67	71	—	—	70	73	—	—	71	74
50	—	—	—	70	—	—	68	72	—	—	69	73
55	—	—	—	68	—	—	67	71	—	—	68	72

be taken to apply to series-parallel equipments employing geared, series-wound, continuous-electricity motors, designed for operation *without* forced draught and proportioned for a load which, averaged over their period of daily service, amounts to 25 per cent. of their 1-hour, 75° C. rated load. Equipments worked up to a higher average

* Among other electric railways which I have investigated in this manner may be mentioned the following:—

Heysham, Morecambe and Lancaster section of the Midland Railway. "Proceedings Institution of Civil Engineers," vol. clxxix. pt. 1, p. 31.

South-Side Elevated Railway of Chicago. "Transactions of the American Institute of Electrical Engineers," vol. xvi. p. 193.

General Electric Company (U.S.A.) Experimental Track. Ibid. vol. xix. p. 831.

New York and Port Chester Railway Company (U.S.A.). Ibid. vol. xix. p. 180.

Grand Rapids, Grand Haven and Muskegon Railway. Ibid. vol. xxiii. p. 691.

Vienna-Baden Railway. Electrical Engineering, December 12, 1907.

load than this, *i.e.* equipments not proportioned to be thermally capable of sustaining for several hours the schedule as regards speed, number of stops, and duration of stops, will, if correctly designed and operated, have higher efficiencies than those indicated in Table XXV., whereas equipments where the average load is less than 25 per cent. of the rated load will have lower efficiencies than those indicated in Table XXV. Equipments employing *gearless* motors will have higher efficiencies than those indicated in Table XXV. Single-phase equipments of the best types yet commercially developed will, for services with high schedule speeds and frequent stops, have only slightly lower efficiencies than those indicated in Table XXV.

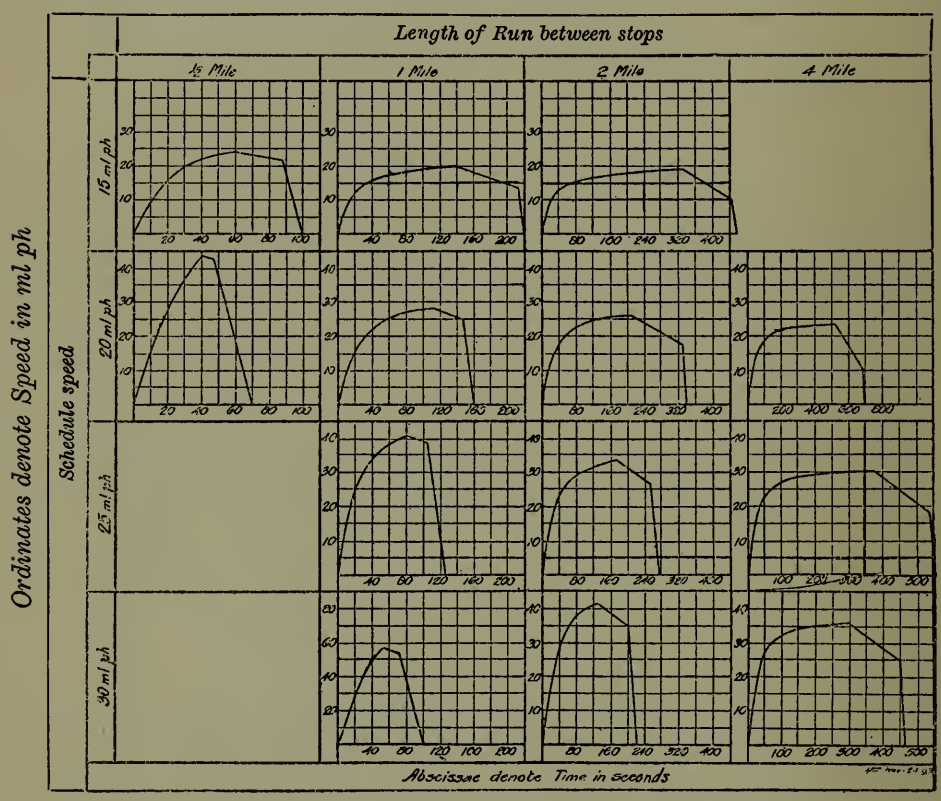


FIG. 44.—Representative Speed-time Diagrams for Various Schedule Speeds and Runs under Working Conditions.

The influence on the over-all efficiency of the average load carried by the equipment during the time it is in circuit is very considerable, as may be seen from the curve in Fig. 60, on p. 105 of Chapter VII. Indeed, in support of the exaggerated claims made for the single-phase motor as applied to severe services (*i.e.* for services for high schedule speeds and frequent stops), tests have been made with electrical equipments which, during the runs, were carrying so great a load that the schedule speed could not have been maintained for

much over two hours without exceeding desirable limits of temperature rise. In view of the fundamental considerations indicated in this chapter, the reader will appreciate that equipments overloaded in this manner will have higher efficiency than when operated at the conservative loads corresponding to sound practice as regards temperature rise. Nevertheless, even under these conditions, the single-phase equipments which I have in mind, namely those in operation on the Heysham Branch of the Midland Railway, only developed over-all efficiencies of the order of 68 per cent. to 72 per cent. There is but little to choose between continuous and single-phase equipments as regards their efficiency under the conditions of actual service, but any superiority in this respect which there may be is certainly possessed by continuous equipments. This cannot be too emphatically pointed out, since utterly unfounded statements have been made to the contrary by various engineers. The reader may be interested in this connection to look up an article on p. 341 of the *Light Railway and Tramway Journal* for June 11, 1909; also a letter in the *Railway Gazette* for July 2, 1909, where he will find typical instances of the unfounded claims to which I allude.

Fig. 44 consists of a chart of speed-time diagrams which are representative of preferable practice for the corresponding schedule speed and number of stops. By employing the data and methods set forth in the preceding chapters, together with the efficiency values given in Table XXV., the estimations set out in Table XXVI. have been made. These estimations relate in the first instance to the energy consumptions at the train, corresponding to the speed-time diagrams of Fig. 44, and at the end of the table estimates of the rated capacity of electrical equipment which should be installed for each schedule are worked out.*

The results obtained in Table XXVI. are plotted in the curves of Figs. 45 to 48.

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* The subject of energy consumption of trains running to different schedules, as also of the motor capacity to be installed, is dealt with by C. T. Hutchinson, in a paper entitled "The Relation of Energy and Motor Capacity to Schedule Speed in the Moving of Trains by Electricity," which was read before the American Institute of Electrical Engineers, *vide* "Transactions," vol. xix. p. 129. See also a paper by W. B. Potter on "The Selection of Electric Motors for Railway Service" (*ibid.* p. 169).

TABLE XXVI.—ESTIMATES OF THE ENERGY CONSUMPTION AND AMOUNT OF EQUIPMENT REQUIRED BY TRAINS OPERATING TO A GIVEN SCHEDULE UNDER NORMAL WORKING CONDITIONS.

1	M, distance from start to stop (miles)	0·5		1·0				2·0				4·0		
2	Schedule speed (ml ph)	15	20	15	20	25	30	15	20	25	30	20	25	30
3	Number of stops per hour	30	40	15	20	25	30	7·5	10	12·5	15	5·0	6·25	7·5
4	Q, duration of each stop (seconds)	20	20	20	20	20	20	20	20	20	20	20	20	20
5	Aggregate duration of stops per hour (seconds)	600	800	300	400	500	600	150	200	250	300	100	125	150
6	Aggregate duration of motion per hour (seconds)	3,000	2,800	3,300	3,200	3,100	3,000	3,450	3,400	3,350	3,300	3,500	3,475	3,450
7	T, time from start to stop (seconds)	100	70	220	160	124	100	460	340	268	220	700	556	460
8	Average speed from start to stop (ml ph)	18·0	25·7	16·4	22·5	29·0	36·0	15·7	21·2	26·9	32·8	20·6	25·9	31·3
9	Crest speed (ml ph). Obtained from the speed-time diagram	24	43·5	20	28	41	57	19	26	33	41	24	30	36
10	Ratio of crest to average speed	1·33	1·62	1·22	1·27	1·42	1·57	1·22	1·23	1·24	1·25	1·16	1·16	1·16
11	Total time during which electricity is on (seconds)	60	40	140	112	80	52	330	220	170	130	520	375	300
12	Mean speed during acceleration (ml ph). Obtained from the speed-time diagram	17·1	25·6	16·8	22·2	28·9	34·6	16·1	21·8	26·6	31	21·5	26·8	31·1
13	Distance covered during acceleration (miles)	0·285	0·285	0·654	0·69	0·64	0·5	1·48	1·27	1·26	1·12	3·2	2·8	2·59
14	Ditto (m)	460	460	1,050	1,110	1,030	804	2,380	2,040	2,020	1,800	5,160	4,500	4,160
15	Train-friction (kg per ton)	6	6	6	6	6	6	6	6	6	6	6	6	6
16	Energy of momentum at crest speed (w hr per ton)	17·4	57·1	12·1	23·6	50·9	98·2	10·9	20·5	33·0	50·9	17·4	27·3	39·1

17	Train-friction energy during time electricity is on (kg m per ton) .	2,760	2,760	6,300	6,660	6,180	4,824	14,280	12,240	12,120	10,800	30,800	27,000	25,000
18	Ditto (w hr per ton) . . .	7·5	7·5	17·2	18·2	16·8	13·1	38·9	33·4	33·0	29·5	84·1	73·6	68·0
19	Total energy per ton to be supplied electrically during each run from start to stop, for purposes of momentum and train-friction	24·9	64·6	29·3	41·8	67·7	111·3	49·8	53·9	66·0	80·4	101·5	100·9	107·1
20	Ditto per ton-mile (*i.e.* output from motors in w hr per ton-mile)	49·8	129·2	29·3	41·8	67·7	111·3	24·9	26·9	33·0	40·2	25·4	25·2	26·8
21	Over-all efficiency of electrical equipment for the run in question (from Table XXV.) (per cent.) .	71	70	75	72	71	69	78	76	74	72	78	77	76
22	Input to train for traction purposes (w hr per ton-mile) . . .	70·6	190	39·3	58·0	97·0	165·0	32·1	35·4	44·6	56·4	32·6	32·8	35·5
23	Portion of the input to the train which is transformed into heat in the electrical equipment (w hr per ton-mile)	20·8	60·8	10·0	16·2	29·3	53·7	7·2	8·5	11·6	14·2	7·2	7·6	8·7
24	Average input over whole run (watts per ton)	1,060	3,860	590	1,160	2,420	4,950	481	708	1,115	1,690	652	820	1,065
25	Average output, ditto . . .	747	2,580	440	836	1,690	3,350	374	540	825	1,205	508	630	804
26	Average output over whole run (hp per ton)	1·00	3·46	0·59	1·12	2·27	4·5	0·5	0·73	1·10	1·62	0·68	0·85	1·08
27	Appropriate 1-hour, 75° C. capacity of motor to be installed in hp per ton of train (taken at 4 times the average output over the whole run)	4·00	13·8	2·36	4·48	8·98	18·0	2·00	2·92	4·40	6·50	2·72	3·40	4·32

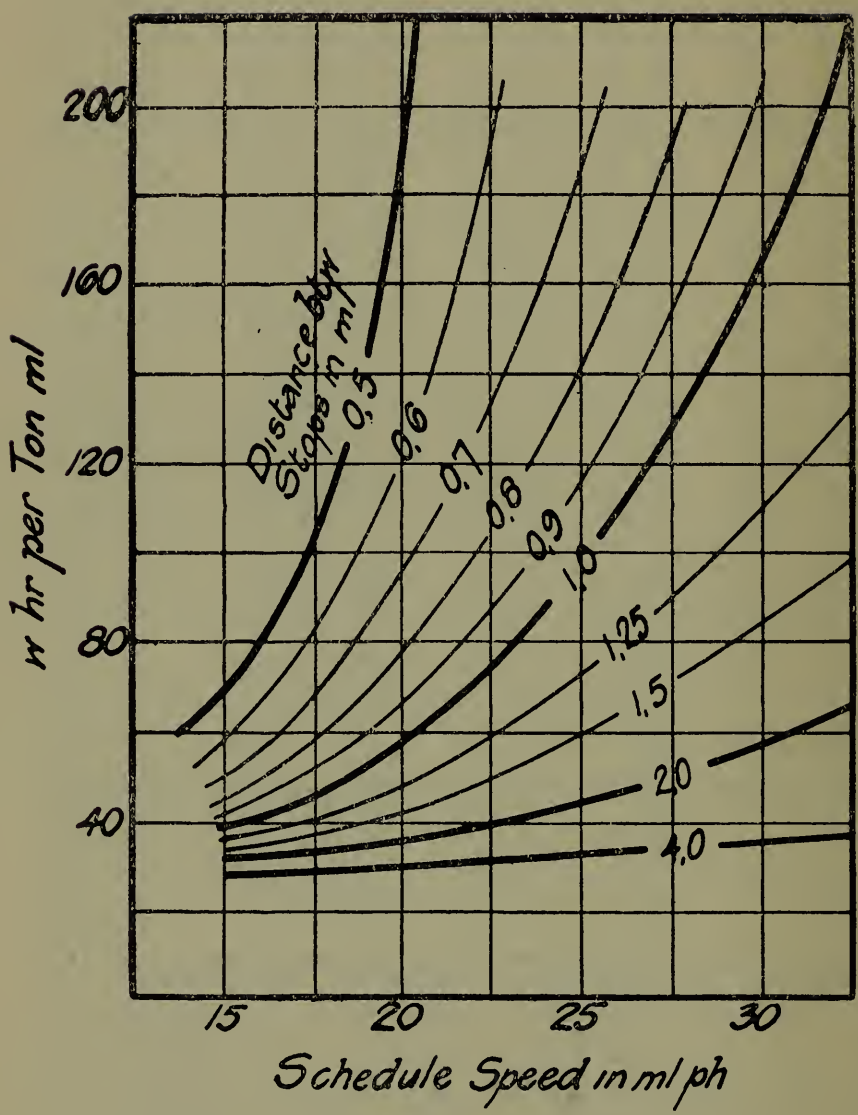

FIG. 45.—Curves giving Conservative Estimates of the Energy Consumption at the Train for Various Schedules, under Normal Working Conditions (from Calculations of Table XXVI., based on the Curves in Fig. 44).

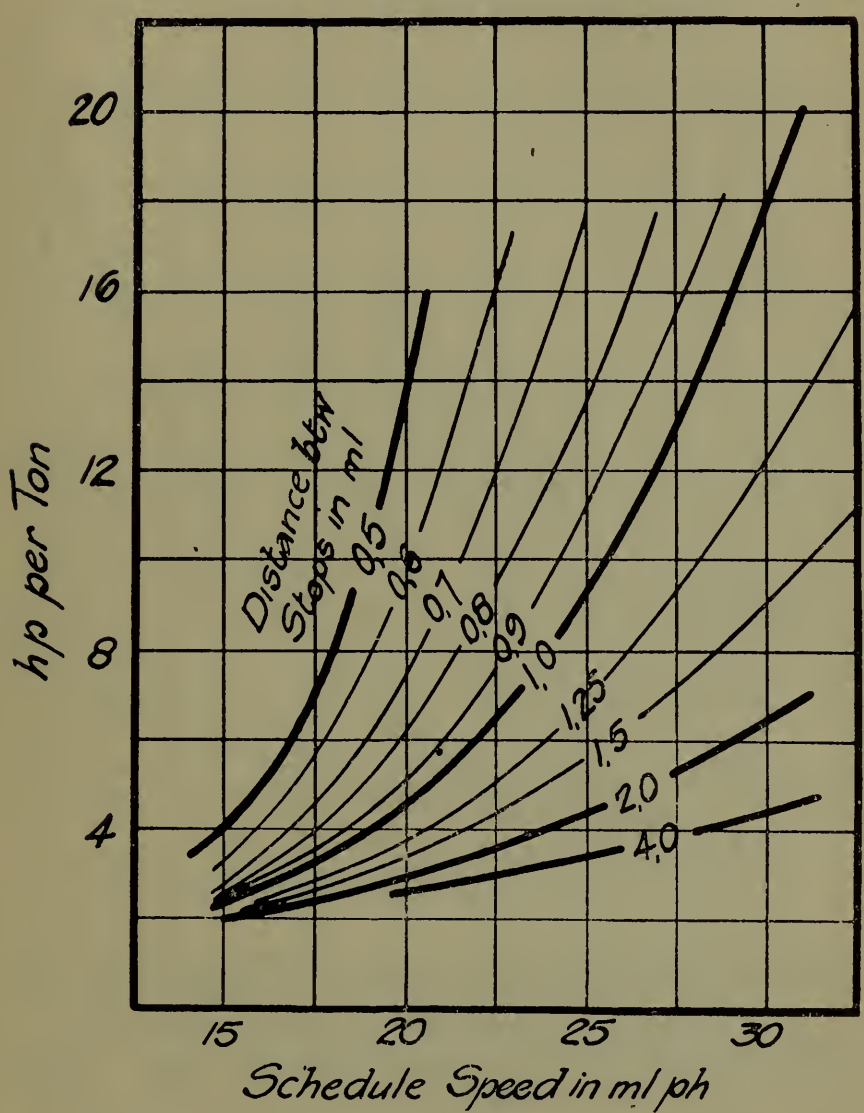

FIG. 46.—Curves giving Conservative Estimates of the Capacity of Equipment in Rated hp of Motors per Ton Weight of Train, necessary for operating Various Schedules under Normal Working Conditions.

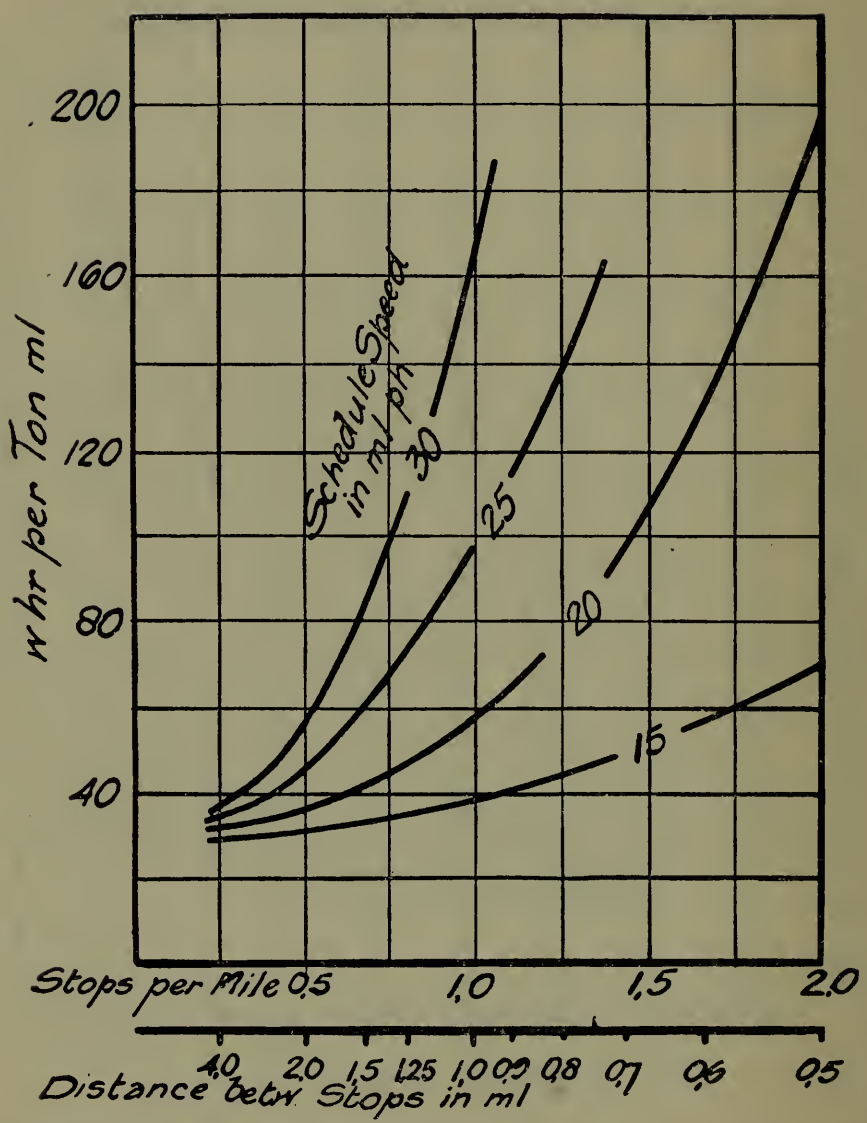

FIG. 47.—Curves giving Conservative Estimates of the Energy Consumption at the Train for Various Schedules, under Normal Working Conditions (from calculations of Table XXVI. based on the Curves in Fig. 44).

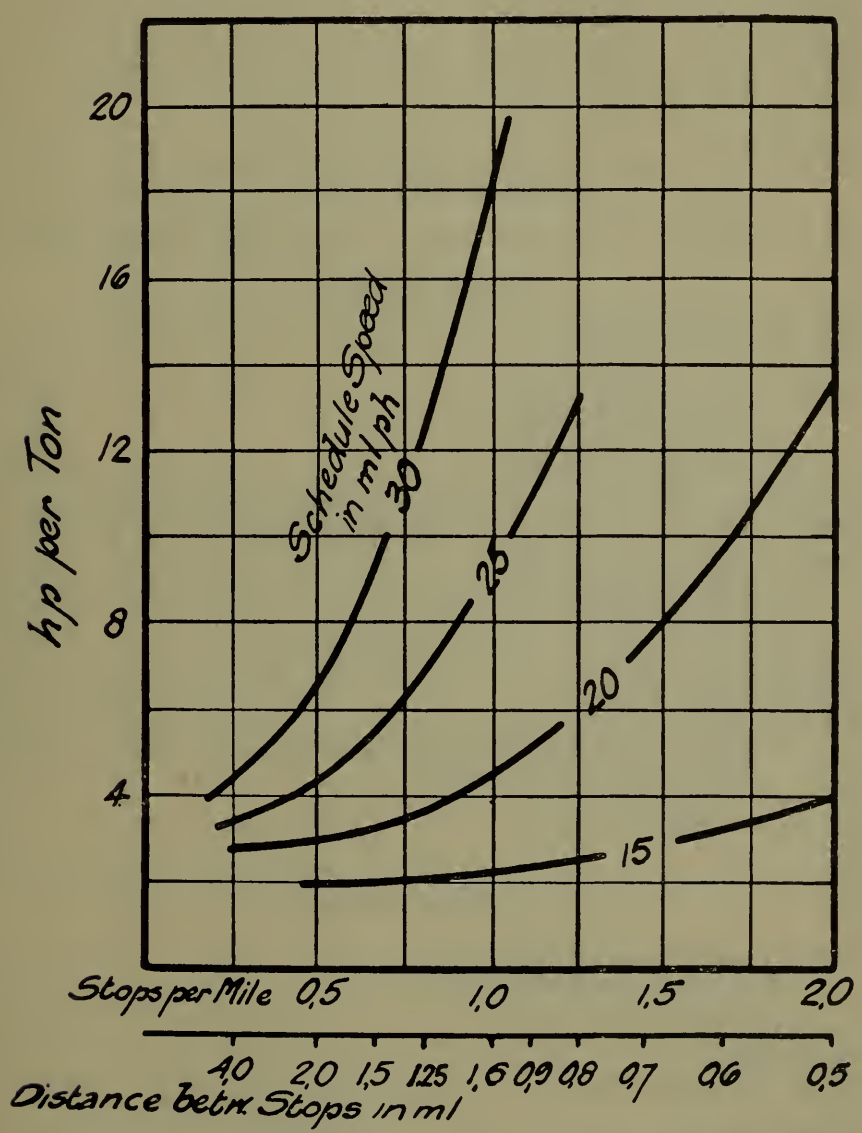

FIG. 48.—Curves giving Conservative Estimates of the Capacity of Equipment in Rated hp of Motors per Ton Weight of Train, necessary for operating Various Schedules under Normal Working Conditions.

Table XXVII. gives a specification for a typical four-coach Lancashire and Yorkshire train, and Fig. 49 is a photograph of the train. This four-coach train is employed both for a stopping service with runs of an average length of 1·32 mile between stations, in which case the schedule speed is 30 ml ph, and for an express service with a schedule speed of 44·5 ml ph, making one intermediate stop in the total run of 18·5 mile.*

TABLE XXVII.—SPECIFICATION OF FOUR-COACH TRAIN ON LANCASHIRE AND YORKSHIRE RAILWAY.

A.—Motor-Coach. B.—Trailer-Coach. C.—Complete Four-Coach Train.

A.—MOTOR-COACH.

General—

Total length over buffers	62 ft 9½ in
,, ,, framework	60 ft 4¾ in
Length between centres of trucks	40 ft 6 in
,, of motor compartment	4 ft 6⅜ in
,, of luggage compartment	5 ft 5⅝ in
,, of passenger compartment	42 ft 5 in

There are two divisions in the passenger compartment, with a door between. The main doors are situated at the ends of the compartment.

Height, over-all, above rail level	12 ft 7⅞ in
,, of driver's cab floor above rail level	4 ft 4¼ in
,, of passenger floor ,, ,,	4 ft 4¼ in
,, of centre of gravity of coach above rail level	3 ft 9 in
Width, over-all, outside	10 ft 0 in
Seating capacity (3rd Class)	69
Weight of coach without passengers	46 tons
,, of car body, including under-frame, air compressors, seats, upholstering, and all fittings	22·5 tons
Seats per foot length of coach	1·14
,, per ton weight of coach	1·5

Trucks—

Both trucks on the motor-coach have each of their axles driven by motors, and are similar.

Weight of motor-truck without motor	6·11 tons
,, of complete motor-truck	12·45 tons
,, sustained per axle on rail	11·5 tons
Wheel-base of truck	8 ft
Gauge of truck	4 ft 8½ in
Diameter of driving wheels	3 ft 6 in

Electrical Equipment—

Type or make of motor	Messrs. Dick-Kerr
Rated hp	150

Method of control is the Dick-Kerr direct system. The two motor-coaches carry the electrical equipment at the ends of the train. A number of coaches are also arranged for Multiple Unit System of control.

* For a description of the electrification of the Lancashire and Yorkshire Railway the reader is referred to Mr. J. A. F. Aspinall's Presidential Address to the Institution of Mechanical Engineers ("Proceedings, Institution of Mechanical Engineers," 1909, No. 2, pp. 423–491).

Particulars of Gear on Motors—

Spur wheel	43 teeth
Pinion	22 teeth
Gear ratio	1·95
Weight of gear with case, about	122 kg
Weight of one motor alone	2·75 tons
,, of motor and gear	2·86 tons
Number of motors per motor-coach	4
Total weight of motors with gearing per motor-coach	11·7 tons
Weight of balance of electrical equipment (*i.e.* controllers, rheostats, etc.) per motor-coach	2·99 tons
Total weight of electrical equipment per motor-coach	14·69 tons
Ratio of total weight of electrical equipment to weight of motors and gearing	1·24
Weight of motor in kg per hp (rated)	18·6
,, of motor and gear in kg per hp (rated)	19·4
,, of motors and gear in kg per ton weight of motor-coach	259
,, of all electrical equipment in kg per ton weight of motor-coach	323
Ratio of total weight of electrical equipment to weight of motor-coach	0·323

B.—Trailer-Coach.

Length over framework	60 ft 0 in
,, between centres of trucks	40 ft 6 in
Total length of passenger compartment	53 ft 0 in
Divisions of passenger compartment	2

Arrangement of doors is similar to motor-coach—at the ends of the compartments.

Height of top of coach above rail level	12 ft 7⅞ in
,, of floor above rail level	4 ft 4¼ in
Width, over-all, outside	10 ft 0 in
Number of seats per coach (alternative)	1st Class, 66 (*a*)
,, ,, ,,	3rd ,, 80 (*b*)
,, ,, ,,	3rd ,, 90 (*c*)
Weight of trailer-coach (*a* and *b*)	26 tons
,, ,, ,, (*c*)	27·58 tons
Seats (1st Class) per foot of length of coach	1·10
,, ,, per ton of weight of coach	2·54

C.—Complete Train.

Number of motor-coaches	2
,, of trailer ,,	2
Total length of train over buffers	248 ft 6 in
,, weight of train without passengers	144 tons
Weight of motor-coach component	92 tons
,, of trailer-coach ,,	52 tons
Total seating capacity	270

The motor-coaches are fitted with third-class seats, while there are trailer-coaches of both first- and third-class seats.

The seating capacities are—

(1) 2 motor-coaches, 3rd class + 2 trailer-coaches, 1st class = 270 seats.
(2) ,, ,, ,, + 1 trailer, 1st class + 1 trailer, 3rd class = 284 seats.
(3) ,, ,, ,, + 2 trailer-coaches, 3rd class = 298 seats.

The first alternative is more general.

Number of motors per train	8
Total hp per train	1200
,, weight of motors and gearing	23·39 tons
Weight of electrical equipment (motor-coach)	29·375 tons
,, ,, ,, (trailer-coach)	0·575 ton
Total weight of electrical equipment per train	29·95 tons
Seats per foot length of train (270 seats)	1·08
,, per ton weight of train	1·87
Rated hp per ton of train	8·33
,, per seat of train	4·44
Total weight of motors and gearing in kg per ton of train	165
,, ,, ,, ,, in kg per seat of train	88·7
,, ,, of electrical equipment in kg per ton of train	210
,, ,, ,, ,, in kg per seat of train	113
Ratio of total weight of electrical equipment to total train weight	0·21

The braking equipment consists of combined automatic, vacuum, and hand systems.

Fig. 49.—Standard 4-Coach Lancashire and Yorkshire Train.

[*To face p.* 86.

CHAPTER VI

THE DETERMINATION OF THE EFFICIENCY OF THE ELECTRICAL EQUIPMENT OF THE TRAINS ON THE CENTRAL LONDON RAILWAY

THE Central London Railway * is built with grades sloping down from the out-going end of the platform, and rising up to the arriving end of the platform. Throughout the length of the platform the track is level. The line originally extended from Shepherd's Bush to the Bank, and is 5·77 miles in length between those two stations. There are 11 intermediate stations, which, together with the two terminal stations—namely, Shepherd's Bush and Bank—make a total of 13 stations. The average distance between stations is—

$$\frac{5{\cdot}77}{12} = 0{\cdot}48 \text{ mile.}$$

The sketch in Fig. 50 is typical of the gradients on a representative section of 0·48 mile (772 m) length. On leaving the

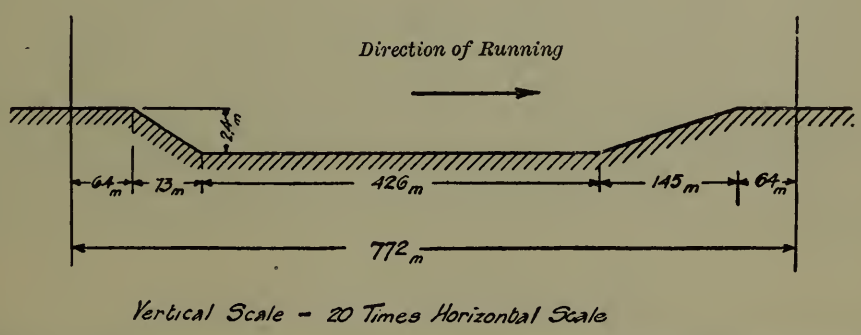

FIG. 50.—Typical Gradients on a Section of 0·48 mile (772 m) on the Central London Railway.

station, every ton of weight of train experiences, during the time that electricity is being absorbed from the supply, a vertical fall

* A very complete description of the Central London Railway, accompanied by 173 illustrations, was published in *Traction and Transmission*, Nos. 29–31, 1903, by H. F. Parshall, E. Parry, and W. Casson.

of 2·4 m. Thus, in addition to the energy transformed from electricity into momentum and into heat, there is a certain amount of energy transformed from potential energy of altitude into momentum and into heat. This latter amount is equal to 2·4 ton-meters, or 2400 kg m or $\left(\frac{2400}{367} = \right)$6·6 w hr per ton weight of train. Since the distance between stops is 0·48 mile, this potential energy of altitude works out at—

$$\frac{6{\cdot}6}{0{\cdot}48} = 14{\cdot}0 \text{ w hr per ton-mile.}$$

By means of wattmeters on the train, measurements have been made of the total amount of electricity consumed during complete round trips from Shepherd's Bush to the Bank and back to Shepherd's Bush. The train, in the course of a complete round trip, started from a siding just beyond Shepherd's Bush station and pulled into Shepherd's Bush, where it stopped for passengers. After discharging its passengers at the Bank, the train ran into a siding just beyond the Bank station, stopped there, and returned to the Bank station, where it stopped to take on passengers. After discharging its passengers at Shepherd's Bush it again pulled into the siding and stopped. A complete round trip thus involved starting the train 28 times. But in 4 out of these 28 starts it was only required to run the train up to a slow speed, and to traverse a very short distance. We may take these four operations as equivalent to one normal run over a 0·48-mile section so far as relates to energy consumption. Thus, we may analyse these round trips as if they had comprised 25 runs, each of 0·48 mile in length. For the purposes of tests, runs were made occupying 22 minutes for the journey from Shepherd's Bush station to the Bank station, and also occupying 22 minutes for the return journey from the Bank station to Shepherd's Bush station. Since the distance each way is 5·77 miles the schedule speed is—

$$\frac{60}{22} \times 5{\cdot}77 = 15{\cdot}7 \text{ ml ph.}$$

The stops between stations were of 15 seconds' duration. A train making a schedule speed of 15·7 ml ph, and with a stop every 0·48 mile, makes stops at the rate of—

$$\frac{15{\cdot}7}{0{\cdot}48} = 32{\cdot}8 \text{ per hr.}$$

Thus, if the train ran continuously to this schedule, the aggregate duration of the 32·8 stops per hour would amount to (32·8 × 15 =)

492 seconds. Consequently, the train would be in motion during (3600 − 492 =)3108 seconds out of the 3600 seconds in each hour.

The time elapsing from start to stop is thus—

$$\frac{3108}{32{\cdot}8} = 95 \text{ seconds.}$$

Thus, we have—

$$T = 95.$$

The average speed, *i.e.* the average speed between stations, is thus—

$$\frac{95 + 15}{95} \times 15{\cdot}7 = 18{\cdot}2 \text{ ml ph.}$$

No exact measurements were made of the crest value of the speed, but the speed-time diagram shown in Fig. 51, drawn to comply with

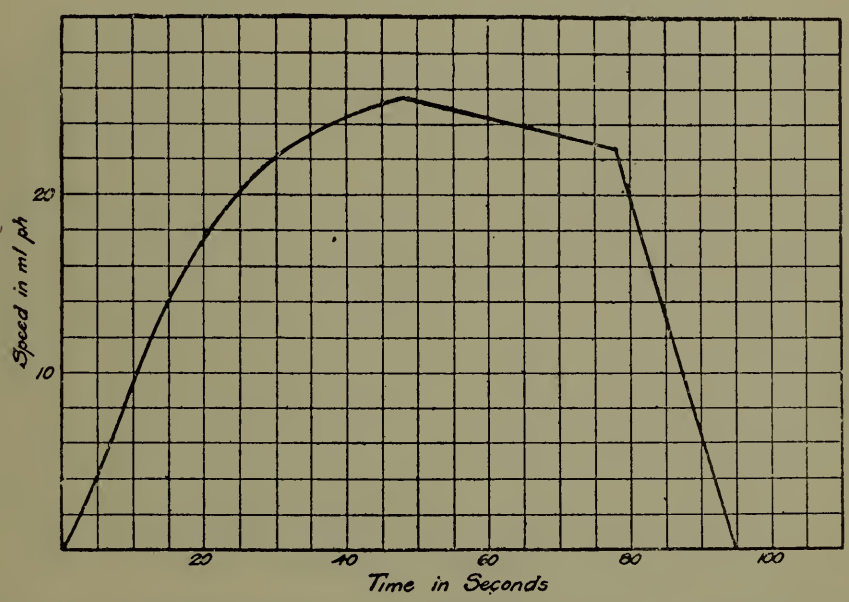

FIG. 51.—Typical Speed-time Diagram for 0·48-mile Run at Schedule Speed of 15·7 ml ph.

the above-stated average speed, and for M = 0·48 and T = 95, must evidently be a close approximation to the truth. The crest speed of this speed-time diagram is 25·5 ml ph. Electricity is supplied to the train during the 48 seconds required to attain this crest speed. At that point the electricity is cut off, and during the remainder of the journey the propulsion-energy is supplied from the momentum of the train. At the crest speed (25·5 ml ph), the momentum of the train is—

$$0{\cdot}0278 \times 1{\cdot}09 \times 25{\cdot}5^2 = 19{\cdot}7 \text{ w hr per ton.}$$

The energy required to supply momentum is thus—

$$\frac{19{\cdot}7}{0{\cdot}48} = 41{\cdot}0 \text{ w hr per ton-mile.}$$

For the first 48 seconds, during which the energy required for overcoming train-friction is obtained from the supply of electricity and from the potential energy of altitude, the average speed is ascertained from Fig. 51, to be 17·1 ml ph. The distance covered during this time is consequently some—

$$\frac{48}{3600} \times 17{\cdot}1 = 0{\cdot}228 \text{ mile} = 366 \text{ m.}$$

Taking the train-friction at 6 kg per ton, our estimate of the energy required for train-friction is—

$$\frac{6 \times 366}{367 \times 0{\cdot}48} = 12{\cdot}5 \text{ w hr per ton-mile.}$$

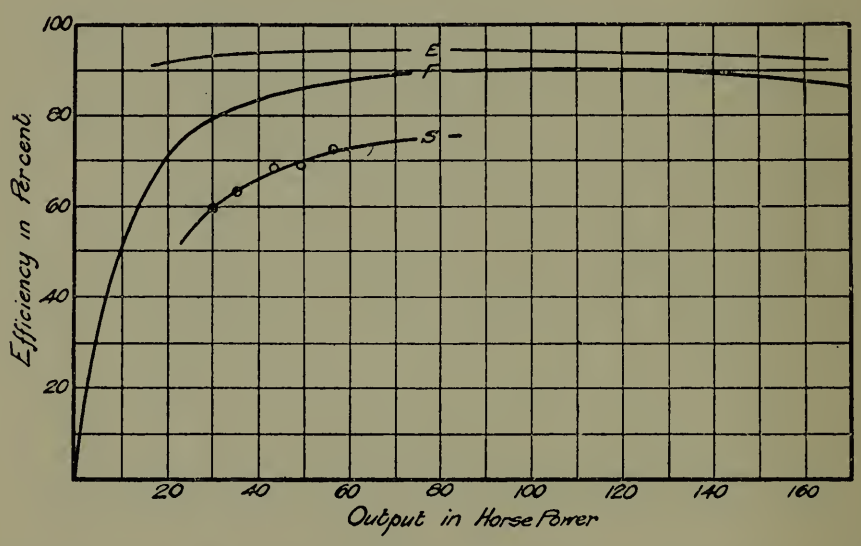

Fig. 52.—Curves of Efficiency at Various Outputs for G.E.66A Motors and Equipment.

Curve E.—G.E.66A motor efficiency (excluding gear).
Curve F.—G.E.66A motor efficiency (including gear) as determined by factory tests at steady load.
Curve S.—Over-all efficiency of the electrical equipment under actual service conditions.

The total energy for momentum and train-friction thus comes to 41·0 + 12·5 = 53·5 w hr per ton-mile. If from this amount we deduct the 14 w hr per ton-mile supplied by the potential energy of altitude, we obtain a remainder of 53·5 − 14·0 = 39·5 w hr per ton-mile as that part of the electrical energy obtained from the supply, which is ultimately transformed into momentum and into the energy

of train-friction. Thus, 39·5 w hr per ton-mile constitutes the *output* of the electrical equipment. Measurements of the *input* to the trains (*i.e.* measurements of the energy consumption) were made on trains composed of various numbers of coaches. The following study relates to the results obtained with trains of from 7 to 3 coaches. The particulars of these trains, together with rough estimates of the train-friction, are given in Table XXVIII. The weight of the average passenger is taken as 62·5 kg (138 lbs). Each motor-coach weighs 23 tons and seats 42 passengers. Each trailer-coach weighs 14 tons and seats 48 passengers.

TABLE XXVIII.—ANALYSIS OF TRAIN TESTS ON THE CENTRAL LONDON RAILWAY.

Number of coaches per train.	7	6	5	4	3
Number of motor-coaches	2	2	2	2	2
„ of trailer-coaches	5	4	3	2	1
Total number of seats	324	276	228	180	132
Weight of empty train (ton)	114	100	87	73	60
Estimated average number of passengers and train staff during test run	80	64	60	54	48
Weight of loaded train (ton)	119	104	91	77	63
Portion of total weight of train allocated to motor-coaches, including passengers (ton)	46	46	47	47	48
Ditto to trailer-coaches, including passengers (ton)	73	58	44	30	15
Added distance allowed for siding operations at the two ends of the line (mile)	0·48	0·48	0·48	0·48	0·48
Total number of runs which are equivalent to a run over the representative section (Fig. 50)	25	25	25	25	25
"Equivalent" total distance (mile)	12·0	12·0	12·0	12·0	12·0
Ton-miles for the round trip	1,430	1,250	1,090	925	756
Total input to train per round trip of 25 "equivalent" runs (w hr)	85,000	77,700	67,800	62,000	53,200
Amount deducted for lighting and for the compressors of the braking equipment (w hr)	7,000	6,000	5,000	4,100	3,100
Total input required for traction per round trip (w hr)	78,000	71,700	62,900	57,900	50,100
W hr per ton-mile of loaded train	54·5	57·4	57·6	62·5	66·2
Portion allocated to momentum and train-friction	39·5	39·5	39·5	39·5	39·5
Portion allocated to losses in the electrical equipment	15·0	17·9	18·1	23·0	26·7
Corresponding efficiency of electrical equipment	72·5	68·9	68·5	63·2	59·6

The gradual decrease in the efficiencies with decreasing numbers of trailers is readily understood when we consider that the average load on the motors is smaller the less the number of trailers, and that the efficiency of the motors falls off rapidly with decreasing load. To further elucidate this point, let us carry through the calculations shown in Table XXIX.

TABLE XXIX.—CONTINUED ANALYSIS OF TRAIN TESTS ON THE CENTRAL LONDON RAILWAY.

Number of coaches per train.	7	6	5	4	3
Number of motor-coaches	2	2	2	2	2
,, of trailer-coaches	5	4	3	2	1
Weight of loaded train (ton)	119	104	91	77	63
Input to traction equipment per ton-mile (w hr)	54·5	57·4	57·6	62·5	66·2
Ditto per train-mile (w hr)	6,490	5,960	5,250	4,810	4,170
Ditto (kw hr per train per hour)	102	93·7	82·5	75·6	65·6
Average rate of consumption of electricity per train by traction equipment (kw)	102	93·7	82·5	75·6	65·6
Pressure at train (volts)	500	500	500	500	500
Average current supplied to train for traction (amperes)	204	187	165	151	131
Number of motors per train	4	4	4	4	4
Current per motor (amperes)	51·0	46·8	41·3	37·8	32·8
Rated output of each motor (hp)	125	125	125	125	125
Efficiency of motor, including gearing at its rated load (per cent.)	90	90	90	90	90
Current input to motor at its rated load (amperes)	208	208	208	208	208
Average current for these tests in per cent. of current at rated load of motors	24·5	22·5	19·8	18·2	15·8
Over-all efficiency of traction equipment for entire run (per cent.)	72·5	69·0	68·4	63·2	59·6
Average load on motors taken over entire run (kw)	74·0	64·5	56·5	47·8	39·1
Ditto (hp)	99·1	86·5	75·7	64·0	52·4
Average load per motor (hp)	24·8	21·6	18·9	16·0	13·1
Percentage which average output constitutes of rated load of motor	19·8	17·3	15·1	12·8	10·5
†Average load on each motor in hp, taken over the time during which electricity is being consumed	56·8	49·5	43·4	35·6	30·0
Percentage which this load constitutes of rated load	45·5	39·6	34·7	28·5	24·0
*Efficiency of motor alone, from factory tests at steady load, for above average load while electricity is being consumed	87·4	86·0	84·5	82·0	79·0
Ratio of efficiency of motor at factory on steady load to efficiency of electrical equipment in service	1·20	1·24	1·24	1·30	1·33

The four 125-hp motors and their control apparatus constitute an aggregate equipment of 500 hp for each of the above trains. This electrical equipment weighs 9·8 tons.

With this added data, let us institute a few further comparisons in Table XXX.

* These values are obtained from the Motor Efficiency Curve given in Fig. 52 for the values of hp per motor given in Item †.

Fig. 53.—Outside Elevation of Central London Motor-Coach.

Fig. 54.—Outside Elevation of Central London Trailer-Coach.

Fig. 55.—Interior View of Central London Trailer-Coach.

[*To face p.* 92.

TABLE XXX.—CONTINUED ANALYSIS OF TRAIN TESTS ON THE CENTRAL LONDON RAILWAY.

Number of coaches per train.	7	6	5	4	3
Weight of loaded train (ton) . . .	119	104	91	77	63
Rated hp per ton weight of train . .	4·20	4·80	5·50	6·50	7·98
Weight of electrical equipment (kg per ton weight of train)	82·4	94·2	108	127	155

This last item, the weight of the electrical equipment per ton weight of train, will vary greatly with the service required as regards the schedule speed and the number of stops per mile.

We have seen, in Table XXIX., that for a 6-coach train operating to a schedule speed of 15·7 ml ph, with a stop every 0·48 mile, the average load per motor during these test runs was only 21·6 hp, which is only 17·3 per cent. of the rated output of the motors. In regular service, with the less skilful manipulation of the average driver, and with the necessity of keeping to the time-table under adverse circumstances, such as occasional prolonged stops in discharging and embarking passengers, the average load will be from 20 to 25 per cent. more of the rated load, instead of the 17·3 per cent. obtaining on test. Under these conditions the temperature-rise of the motor after its day's work of from 15 to 18 hours' consecutive service, is some 60°. The equipments are quite ample for handling 7-coach trains, but the traffic appears to be well met with 6-coach trains made up of two motor-coaches and four trailers in the busy hours. During the middle of the day, 3-coach trains, consisting of one motor-coach and two trailers, are sometimes substituted.

While the schedule speed of 15·7 ml ph has been employed on occasions, with a running time of 22 minutes from Shepherd's Bush to the Bank, the customary schedule is 14 ml ph, which corresponds to a time of 25 minutes between Shepherd's Bush and the Bank.

Table XXXI. consists of a specification of this Central London Railway 6-coach train. Fig. 53 shows a photograph of the motor-coach complying with this specification, and Fig. 54 a photograph of the trailer-coach. A photograph of the inside of this Central London trailer-coach is shown in Fig. 55.

The Central London Railway now extends beyond Shepherd's Bush to Wood Lane, and at the other end it is about to be extended beyond the Bank to Liverpool Street. This latter section was included in the preliminary scheme for the line, but, for various considerations, was not constructed.

TABLE XXXI.—SPECIFICATION OF A SIX-COACH TRAIN ON THE CENTRAL LONDON RAILWAY.

A.—Motor-Coach. B.—Trailer-Coach. C.—Complete Train.

A.—MOTOR-COACH.

General—

Total length of coach over headstocks	45 ft 6 in
Length of driver's cab (and equipment compartment)	12 ft 0 in
,, of passenger compartment	30 ft 3 in
,, of rear platform	3 ft 3 in
Length between centres of bogies	29 ft 0 in
Height, over-all, from rail	9 ft 4½ in
,, of passenger floor above rail	2 ft 0 in
Width, over-all, outside	8 ft 6 in
Seating capacity	42
Weight of coach without passengers	23·5 tons
,, on driving wheels without passengers	16·5 tons
,, on trailing wheels without passengers	7·0 tons
,, of coach body, including under-frame, air compressors, seats, upholstering and all fittings	12·2 tons
Seats per foot length of coach	0·92
,, ton weight of coach	1·79

Bogie Trucks—

Weight of motor bogie without motor	3·9 tons
,, trailing bogie, about	2·0 tons
Wheel base of motor bogie	6 ft
,, of trailing bogie	5 ft
Length over frame of motor bogie	9 ft 6 in
,, inside frame of motor bogie	8 ft 10 in
Width over frame ,, ,,	6 ft 6 in
,, inside frame ,, ,,	5 ft 10 in
Gauge of track	4 ft 8½ in
Weight sustained per motor axle (unloaded coach)	8·3 tons
Ditto (portion which is spring supported)	6·3 tons
Ditto (portion which is direct supported)	2·0 tons
Weight sustained per trailing axle (unloaded coach)	3·5 tons
Ditto (portion which is spring supported), about	3·0 tons
Ditto (portion which is direct supported), about	0·5 tons
Diameter of driving wheels (with new tyres)	2 ft 11 in
,, of trailing wheels (,, ,,)	2 ft 5 in
Axle diameter (motor bogie) mid frame	5 in
,, ,, ,, at hub	6 in
,, ,, ,, at gear wheel seat	6 in
,, ,, ,, at journals	4⅝ in

Electrical Equipment —

Type of motor	G.E.66A
Rated hp	125
Pressure in volts	500
Efficiency (per cent.) full load	90
,, ,, ¾ ,,	90
,, ,, ½ ,,	88
,, ,, ¼ ,,	80

The method of control is by the Multiple Unit System, there being two motor-coaches per train. The electrical equipment is carried at the two ends of the train. The contactors and rheostats are placed in a steel compartment (behind the driver's cab) which is separated from the passenger compartment by a steel bulkhead and a lining of asbestos.

Particulars of gear on motor—	
Cast steel spur wheel	59 teeth
Mild steel pinion	15 teeth
Ratio	3·94
Weight (with case), about	210 kg
Weight of one motor alone	1·75 tons
Weight of one motor and gear	1·96 tons
Motors per motor-coach (and per motor bogie)	2
Total weight of motors with gearing per motor-coach	3·92 tons
Weight of balance of electrical equipment (*i.e.* controllers, contactors, rheostats, etc.) per motor-coach	1·0
Total weight of electrical equipment per motor-coach	4·92 tons
Ratio of total weight of electrical equipment to weight of motors and gearing	1·26
Weight of motor in kg per hp (rated)	14·2
,, ,, and gear in kg per hp (rated)	15·9
,, of motors and gearing in kg per ton weight of motor-coach	167
,, of all electrical equipment in kg per ton weight of motor-coach	210
Ratio of total weight of electrical equipment to total weight of motor-coach	0·21

B.—Trailer-Coach.

Total length of coach over headstocks	45 ft 6 in
Length of passenger compartment	39 ft
,, of rear platforms, each	3 ft 3 in
,, between centres of bogies	29 ft 6 in
Height, over-all, from rail	9 ft 4½ in
,, of floor above rail	2 ft 0 in
Width, over-all, outside	8 ft 6 in
Wheel base of bogies	5 ft 0 in
Seating capacity	48
Weight of coach without passengers	13·5 tons
Seats per foot length of coach	1·05
,, ton weight of coach	3·56

The doorways are 2 ft 10 in wide, and are situated at the ends of the passenger compartments.

C.—Complete 100-Ton, 6-Coach Train.

Number of motor-coaches	2
,, of trailer-coaches	4
Total length	276 ft
Weight of motor-coach component	47 tons
,, of trailer-coach ,,	54 tons
Total weight of 6-coach train	101 tons
,, seating capacity	276
Number of motors on train	4
Total hp per train (rated)	500
Weight of motors and gearing per train	7·84 tons
Total weight of electrial equipment	9·8 tons
Seats per foot length of train	1·00
,, ton weight of train	2·73
Rated hp per ton of train	4·95
,, ,, seat of train	1·81
Total weight of motors and gearing in kg per ton weight of train	78
,, ,, electrical equipment in kg per ton weight of train	98
Ratio of total weight of electrical equipment to total weight of train	0·098

The braking equipment consists of Westinghouse Automatic Brakes, for which motor-driven compressors and the necessary brake gear are provided beneath the coaches.

Examples.

1. A railway line has its profile similar to Fig. 50, but with a vertical fall of 4 m, and a distance between stations of 0·68 mile (1290 m). A train is run over the line at a schedule speed of 20 ml ph. Estimate the actual train consumption.

Ans. 84 w hr per ton-mile (100 w hr per ton-mile on *level* track).

2. From Table XXVIII. plot the values of the energy input (w hr per ton-mile) as ordinates against train weights as abscissae, and notice the marked decrease in train consumption per ton-mile with increased train weight.

3. As determined from the speed-time diagram for a schedule speed of 25 ml ph and 1·2 mile between stops, the input for momentum and friction was 64 w hr per ton.

Mean acceleration and deceleration was 1·5 ml phps.

Estimate (*a*) Probable input per ton-mile.
(*b*) Average input per ton.

Ans. (*a*) 75 w hr per ton-mile.
(*b*) 1·88 kw per ton.

4. A train weighing 130 tons is to run to a schedule of 20 ml ph, 0·6 mile between stops. (*a*) What motors should be installed? (*b*) If the electricity is supplied for two-thirds of the distance, and train friction is 6 kg per ton, estimate the crest speed to be attained during each run if the efficiency of the equipment is 67 per cent.

Ans. (*a*) Eight 160-hp motors.
(*b*) 33·5 ml ph.

CHAPTER VII

ANALYSIS OF SOME ENERGY CONSUMPTION TESTS OF TRAINS ON THE GREAT NORTHERN PICCADILLY AND BROMPTON RAILWAY

THE tests described in this chapter were made in March, 1907, on a train consisting of one motor-coach and three trailer-coaches. Each motor-coach weighed 27·5 tons, and each trailer weighed 16·5 tons, the weight of train, exclusive of passengers, thus aggregating 77 tons. The weight of the passengers on the train during these tests was estimated to average 3 tons, bringing up the weight of train, inclusive of passengers, to 80 tons.

The train was run over the whole route, and the energy consumed at the train was recorded at each stop. The entire distance between the termini at Hammersmith and Finsbury Park is 8·9 miles, and comprises 20 runs between stations. Thus, M, the average distance from start to stop, is—

$$\frac{8{\cdot}9}{20} = 0{\cdot}445 \text{ mile.}$$

Two tests, A and B, were carried out. Each comprised two runs between termini, in opposite directions, and the average value of the schedule speeds in both directions was—

For Test A . . .	14·56 ml ph.
„ B . . .	15·43 „

The average duration of stops at stations was 12 seconds. The running time between termini was—

For Test A: $\frac{8{\cdot}9}{14{\cdot}56} \times 3600 = 2200$ seconds.

„ B: $\frac{8{\cdot}9}{15{\cdot}43} \times 3600 = 2075$ „

The 19 intermediate stops occupied an aggregate of—

$$19 \times 12 = 228 \text{ seconds.}$$

Consequently, the total time during which the train was in motion was—

For Test A . . . 2200 − 228 = 1972 seconds.
„ B . . . 2075 − 228 = 1847 „

This gives for value of T, the time occupied by the average run from start to stop—

For Test A $\frac{1972}{20} = 98{\cdot}6$ seconds.

„ B $\frac{1847}{20} = 92{\cdot}4$ „

The average speed from start to stop is—

For Test A: $\frac{98{\cdot}6 + 12}{98{\cdot}6} \times 14{\cdot}56 = \frac{110{\cdot}6 \times 14{\cdot}56}{98{\cdot}6} = 16{\cdot}3$ ml ph.

„ B: $\frac{92{\cdot}4 + 12}{92{\cdot}4} \times 15{\cdot}43 = \frac{104{\cdot}4 \times 15{\cdot}43}{92{\cdot}4} = 17{\cdot}5$ ml ph.

The average time during which electricity was supplied to the train was—

For Test A . . . 41 seconds.
„ B . . . 44 „

The maximum speeds were not recorded, but the two speed-time diagrams shown in Fig. 56 may reasonably be taken as approximating

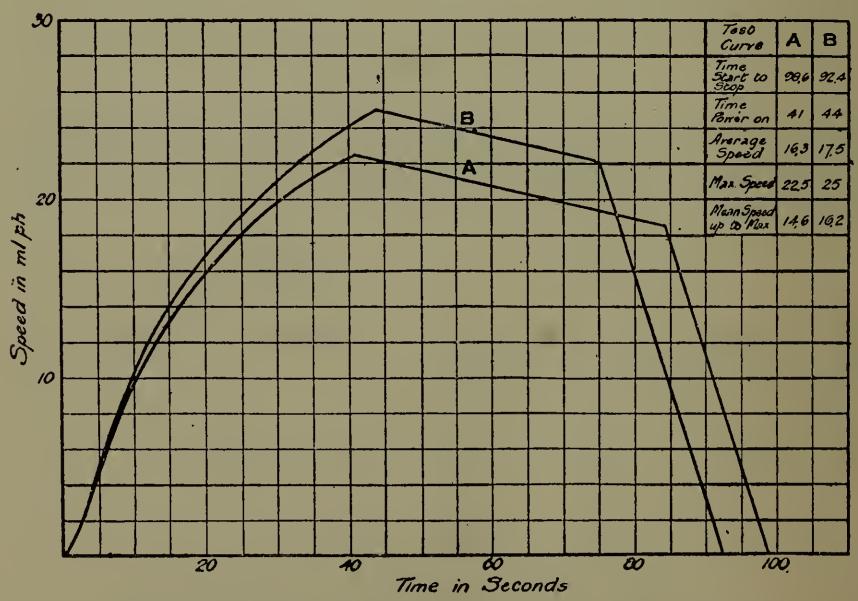

FIG. 56.—Average Speed-time Diagrams for Great Northern Piccadilly and Brompton Railway Tests A and B, from Values in Table XXXII.

to the average conditions for these runs. From these speed-time diagrams we obtain—

For Test A . . . maximum speed = 22·5 ml ph.
„ B . . . „ = 25·0 ml ph.

The altitude of the line at Finsbury Park is considerably greater than at Hammersmith; but by averaging the energy consumption in both directions, a rough approximation to the conditions of a level run is obtained. The average results for Tests A and B are set forth in Table XXXII. on the following page.

The longest run between stops is the 0·99 mile between Baron's Court and Earl's Court. The shortest is the 0·18 mile between Leicester Square and Covent Garden. The average distance between stops over the entire route is, as already stated, 0·45 mile. It is instructive to compare the results for these three cases, averaged over the two directions, to eliminate grades so far as practicable. The three distances are 0·18 mile, 0·45 mile, and 0·99 mile. The average speeds from start to stop are 12·7 ml ph, 16·9 ml ph, and 20·8 ml ph. The amounts of energy consumed at the train work out at 121 w hr per ton-mile, 78 w hr per ton-mile, and 59 w hr per ton-mile. Taking the duration of each stop at 12 sec, the corresponding schedule speeds are 10·2 ml ph, 15·0 ml ph, and 19·5 ml ph. Thus we have the following summary:—

Distance between stops (mile).	Schedule speed (ml ph).	Energy consumption (w hr per ton-mile).
0·18	10·2	121
0·45	15·0	78
0·99	19·5	59

Although the schedule speed in the last case (*i.e.* the 0·99 mile run) is practically twice as great as in the first case (*i.e.* the 0·18 mile run), the energy consumption per ton-mile is only half as great. This is a striking example of the influence of the number of stops, which are 6 per mile in the first case and just about 1 per mile in the last.

TABLE XXXII.—AVERAGE VALUES OBSERVED DURING TWO TRAIN TESTS ON THE GREAT NORTHERN PICCADILLY AND BROMPTON RAILWAY.

	Run between	Distance (mile).	Test A.					Test B.				
			Time that power was on (seconds).	Time from start to stop (seconds).	Average speed from start to stop (ml ph).	Energy consumption (w hr).		Time that power was on (seconds).	Time from start to stop (seconds).	Average speed from start to stop (ml ph).	Energy consumption (w hr).	
						Total for run.	Per ton-mile.				Total for run.	Per ton-mile.
1	Hammersmith—Baron's Court	0·451	54·5	105·5	15·3	3020	82·6	51·5	94·0	17·3	2680	74·3
2	Baron's Court—Earl's Court	0·987	92·5	180·5	19·7	4480	55·4	91·5	163·0	21·8	5070	63·4
3	Earl's Court—Gloucester Road	0·523	43·5	105·5	17·9	2820	66·3	45·5	100·0	18·9	3030	72·5
4	Gloucester Road—South Kensington	0·424	46·0	94·5	16·2	3040	88·5	51·5	87·5	17·5	3570	105·1
5	South Kensington—Brompton Road	0·343	24·5	93·5	13·2	1740	62·7	26·0	88·5	14·0	1870	68·2
6	Brompton Road—Knightsbridge	0·426	33·5	94·5	15·2	2370	68·7	47·0	88·0	17·4	2980	87·5
7	Knightsbridge—Hyde Park Corner	0·342	29·0	83·0	14·9	2100	75·7	32·5	80·5	15·3	2490	90·7
8	Hyde Park Corner—Down Street	0·307	31·0	76·5	14·5	2000	80·6	33·5	71·0	15·6	2380	97·1
9	Down Street—Dover Street	0·342	26·0	82·5	14·9	1850	66·7	27·0	81·0	15·2	2050	74·8
10	Dover Street—Piccadilly Circus	0·344	35·0	77·0	16·1	2520	90·2	37·0	76·5	16·2	2800	101·5
11	Piccadilly Circus—Leicester Square	0·312	25·5	79·0	14·2	1850	73·2	28·0	77·0	14·6	2010	80·6
12	Leicester Square—Covent Garden	0·176	23·5	51·0	12·5	1550	108·8	25·0	49·5	12·8	1880	133·8
13	Covent Garden—Holborn	0·354	47·0	88·5	14·4	2980	103·5	49·0	88·0	14·5	3060	107·5
14	Holborn—Russell Square	0·460	39·5	100·0	16·6	2480	61·4	45·5	96·5	17·2	2850	77·1
15	Russell Square—King's Cross	0·571	45·0	116·5	17·6	3060	66·1	50·0	109·0	18·9	3670	80·3
16	King's Cross—York Road	0·507	45·5	103·5	17·7	3020	73·8	50·5	98·0	18·7	3740	92·5
17	York Road—Caledonian Road	0·731	66·5	138·0	19·1	4130	69·5	69·5	131·5	20·0	4760	81·1
18	Caledonian Road—Holloway	0·388	32·0	88·0	15·9	2250	71·4	35·5	82·0	17·1	2870	92·1
19	Holloway—Gillespie Road	0·451	36·5	98·5	16·5	2640	72·5	40·0	92·5	17·6	3460	96·4
20	Gillespie Road—Finsbury Park	0·456	41·0	99·0	16·6	2230	60·6	41·0	92·0	17·8	2750	75·5
	Averages	0·445	40·9	98·6	16·3	2600	72·4	43·8	92·4	17·5	2900	84·2

Forty-nine per cent. of the run over this line is on curves, and for this reason the estimates of the train-friction loss are based on a tractive resistance of 8 kg per ton.*

With these data the calculations of Table XXXIII. may be made—

TABLE XXXIII.—ANALYSIS OF TRAIN TESTS ON THE GREAT NORTHERN PICCADILLY AND BROMPTON RAILWAY.

Designation of Test.	A	B
Mean length of run between stations (mile)	0·445	0·445
Duration of stop (seconds)	12	12
Time from start to stop (seconds)	98·6	92·4
,, electricity is supplied (seconds)	41	44
Average speed (ml ph)	16·3	17·5
Schedule ,, ,,	14·56	15·43
Input for traction (w hr per ton-mile)	72·4	84·2
Weight of train, including passengers (tons)	80	80
Input for traction (w hr per train-mile)	5790	6740
Kw hr per hr, *i.e.* average kw input	84·4	104
Number of motors per train	2	2
Average kw input per motor	42·2	52·0
Maximum speed (ml ph), from Fig. 56	22·5	25·0
Momentum energy per ton = $0{\cdot}0278 \times 1{\cdot}09 \times V^2$ (w hr)	15·3	18·9
Ditto (w hr per ton-mile)	34·4	42·5
Train-friction (kg per ton)	8	8
Mean speed during time electricity is supplied (ml ph)	14·6	16·2
Distance covered during this period (m)	267	318
Train-friction energy during this period (w hr per ton)	5·81	6·94
,, ,, ,, (w hr per ton-mile)	13·0	15·6
Output for momentum and train-friction (w hr per ton-mile)	47·4	58·1
Residue of input to ascribe to losses in the electrical equipment (w hr per ton-mile)	25·0	26·1
Resulting over-all efficiency of the electrical equipment	65·5	69·0
Average kw output per motor (from kw input and efficiency)	27·6	35·8
,, hp ,, ,, (from kw output)	37·0	48·0
,, hp ,, ,, in per cent. of rated output of motor (200 hp)	18·5	24·0
,, hp output of motor during time electricity is supplied	100	114
Percentage which this load constitutes of rated load	50·0	57·0
Efficiency of the motor on factory test at this load, from Fig. 57	89·2	89·5
Ratio of factory test efficiency of motor to over-all efficiency of electrical equipment	1·36	1·30

In Fig. 57 is plotted the efficiency curve (including gearing) of this motor when operated from a 500-volt circuit under the conditions of the manufacturer's tests at the works. Below the curve are the points corresponding to the efficiency of the electrical equipment as estimated from tests A and B as above described.

* The subject of the influence of curves on the tractive effort, is briefly stated in Chapter IX.

The motors used on the trains tested are of the G.E.69B type, and their weights are given in the detailed specification of the trains,

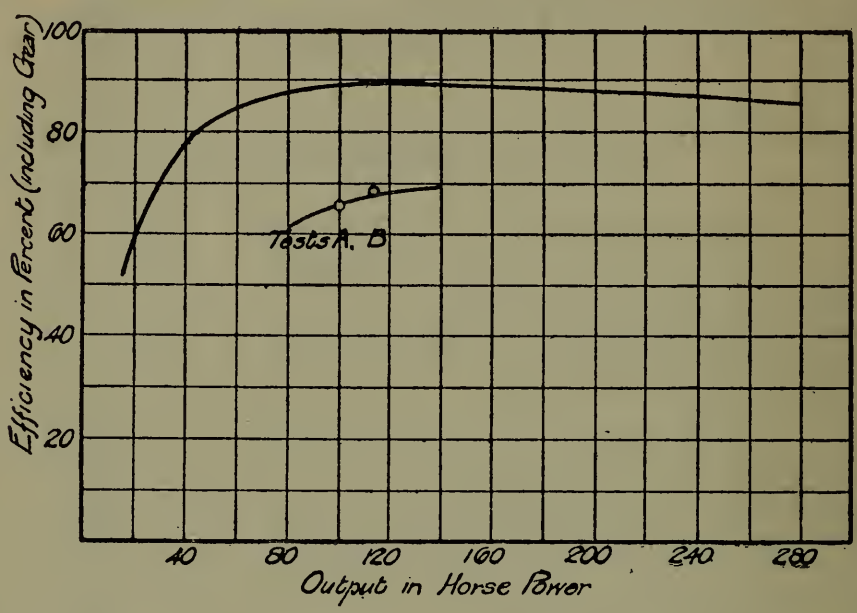

FIG. 57.—Efficiency Curve for G.E.69B Railway Motor, 500 volt, 200 hp, as used on Great Northern Piccadilly and Brompton Railway.

compiled in Table XXXIV. A photograph of the outside of the trailer-coach is shown in Fig. 58, and the general appearance of the inside of the coach is shown in Fig. 59.

TABLE XXXIV.—SPECIFICATION OF SIX AND FOUR-COACH TRAINS ON GREAT NORTHERN PICCADILLY AND BROMPTON RAILWAY.

A.—Motor-coach. B.—Trailer-coach. C.—Complete Train (C1, Six-coach; C2, Four-coach).

A.—MOTOR-COACH.

General—	
Length of coach, over-all	50 ft
,, of coach, over platforms	49 ft
,, of driver's cab (and equipment compartment)	13 ft 8 in
,, of passenger compartment	31 ft 6 in
,, of rear platform	3 ft 8 in
,, between centre of bogies	33 ft
Height, over-all, from rail	9 ft 6½ in
,, of passenger floor above rail	1 ft 11 in
Width, overall, outside	8 ft 8½ in
Seating capacity	46
Weight of coach without passengers	27·5 tons
,, ,, body including under-frame, air compressors, seats, upholstering and all fittings	12·4 tons
Seats per foot length of coach	0·92
,, ton weight of coach	1·67

Bogies and Trucks—

Weight of trailing bogie, about	3 tons
Wheel base of motor bogie	6 ft 6 in
,, of trailing bogie	5 ft 0 in
Motor bogie, length over frame	9 ft 10½ in
,, length inside frame	9 ft 6 in
,, width over frame	6 ft 9 in
,, width inside frame	6 ft 0 in
Trailing bogie, length over frame	8 ft 0 in
,, width over frame	6 ft 6 in
,, width inside frame	5 ft 9 in
Gauge of track	4 ft 8½ in
Diameter of driving wheels	3 ft 6 in
,, of trailing wheels	2 ft 6 in

Electrical Equipment—

Type of motor	G.E.69B
Rated horse-power	200–240

The manufacturer's usual rating is 200 hp, but on the generally accepted rating for railway motors, *i.e.* 1 hour, 75° C. basis, the motor develops 240 hp.

Pressure in volts	500
Efficiency, full load (200 hp) in per cent.	88·0
,, ¾ ,, ,, ,,	89·0
,, ½ ,, ,, ,,	89·2
,, ¼ ,, ,, ,,	82·0

Method of control is by B.T.H. Multiple-Unit System, there being two motor-coaches in the six-coach train, and sometimes one and sometimes two in the four-coach train. The contactors and rheostats are placed in a steel compartment (behind the driver's cab).

Gear ratio	3·2
Weight (with gear case), about	270 kg
,, of one motor alone	2·51 tons
,, of one motor and gear	2·80 tons
Motors per motor-coach (and per motor bogie)	2
Total weight of motors and gearing per motor-coach	5·60 tons
Weight of balance of electrical equipment (*i.e.* controllers, contactors, rheostats, etc.) per motor-coach	1·68 tons
Total weight of electrical equipment per motor-coach	7·28 tons
Ratio of total weight of electrical equipment to weight of motors and gearing	1·3
Weight of motor in kg per hp (200-hp rating)	12·7
,, ,, ,, ,, (240-hp rating)	10·6
,, ,, and gear in kg per hp (200-hp rating)	14·2
,, ,, ,, ,, ,, (240-hp rating)	11·8
,, of motors and gearing in kg per ton weight of motor-coach	207
,, of all electrical equipment in kg per ton weight of motor-coach	270
Ratio of total weight of electrical equipment to total weight of motor-coach	0·27

B.—Trailer-Coach.

Length, over-all	50 ft
,, of coach, over platforms	49 ft
,, of passenger compartment	41 ft
,, of rear platforms	3 ft 8 in
,, between centres of bogies	33 ft
Height, over-all, from rail	9 ft 6½ in
,, of floor above rail	1 ft 11 in
Width, over-all, outside	8 ft 8½ in
Wheel base of trucks	5 ft 0 in

Weight of trucks (two)	5·9 tons
,, of body	10·3 tons
Seating capacity	52
Weight of coach without passengers	16·2 tons
Seats per foot length of coach	1·04
,, ton weight of coach	3·20

C1.—Complete Six-Coach Train.

Number of motor-coaches	2
,, of trailer-coaches	4
Total length of train	300 ft
Weight of motor-coach component	55 tons
,, of trailer-coach ,,	65 tons
Total weight of train	120 tons
,, seating capacity	300
Number of motors on train	4
Total hp per train (motors, 200-hp rating)	800
,, ,, (motors, 240-hp rating)	960
Weight of motors and gearing per train	11·2 tons
Total weight of electrical equipment	14·6 tons
Seats per foot length of train	1·0
,, ton weight of train	2·5
Rated hp per ton of train (motors, 200-hp rating)	6·66
,, ,, ,, (motors, 240-hp rating)	8·0
Rated hp per seat of train (motors, 200-hp rating)	2·66
,, ,, ,, (motors, 240-hp rating)	3·20
Total weight of motors and gearing in kg per ton of train	94·7
,, ,, of electrical equipment in kg per ton of train	122
Ratio of total weight of electrical equipment to total weight of train	0·122

The six-coach train is capable of division into two three-coach trains, which will have the same proportional values per ton and per seat of train as above. Extra master controllers are fitted on the centre trailer-coaches for use when divided into three-coach trains. Many of these three-coach (divided) trains are used during light-service hours, also some four-coach trains (two motor-coaches and two trailer-coaches).

C2.—Complete Four-Coach Train.

Number of motor-coaches	1	2
Number of trailer-coaches	3	2
Total length	200 ft	200 ft
Weight of motor-coach component	27·5 tons	55 tons
Weight of trailer-coach component	49 tons	32·5 tons
Total weight of train	76·5	87·5 tons
Total seating capacity	202	196
Number of motors per train	2	4
Total hp per train (motors, 200-hp rating)	400	800
,, ,, (motors, 240-hp rating	480	960
Weight of motors and gearing per train	5·6 tons	11·2 tons
Total weight of electrical equipment	7·3 tons	14·6 tons
Seats per foot length of train	1·01	0·98
Seats per ton weight of train	2·64	2·24
Rated hp per ton of train (motors, 200-hp rating)	5·2	9·2
,, ,, ,, (motors, 240-hp rating)	6·3	11·0
Rated hp per seat of train (motors, 200-hp rating)	2·0	4·1
,, ,, ,, (motors, 240-hp rating)	2·4	4·9
Total weight of motors and gearing in kg per ton of train	74·4	130
Total weight of electrical equipment in kg per ton of train	96·8	169
Ratio of total weight of electrical equipment to total weight of train	0·097	0·169

Fig. 58.—Outside Elevation of Great Northern Piccadilly and Brompton Railway Trailer-Coach.

Fig. 59.—Interior View of Great Northern Piccadilly and Brompton Railway Trailer-Coach.

[*To face p.* 104.

The reader will not have failed to notice that not only in this G.N.P. and B. train, but also in the C.L.R. trains, and in the L. and Y. trains, the efficiency of the electrical equipment is higher, the greater the percentage of rated load carried by the motors during the time that electricity is being consumed from the line, to supply the electrical equipment on the train. In Fig. 60 the results already given of efficiency estimates for these three railways are brought together. The two results represented by small squares relate to the G.N.P. and B. tests which we have just analysed in this

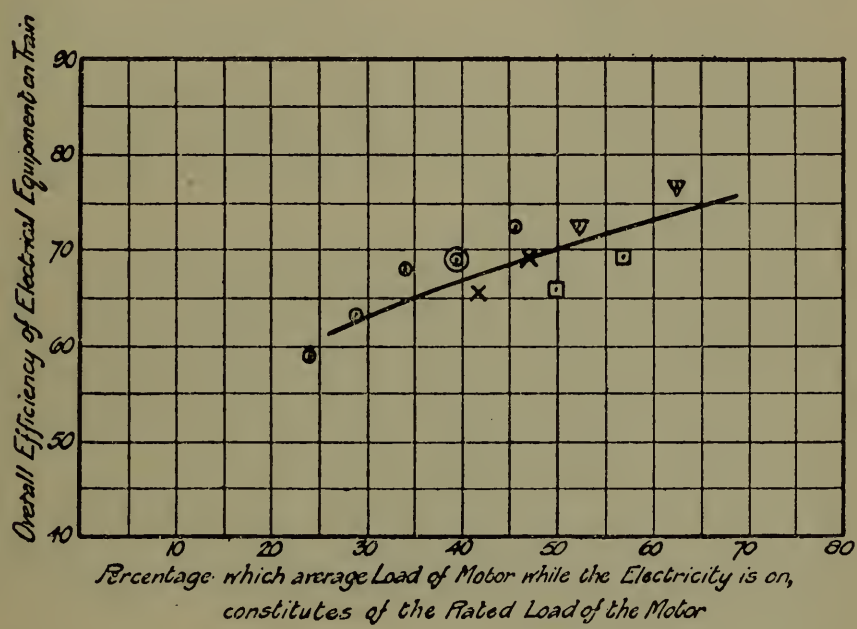

FIG. 60.—Grouping of the Over-all Efficiencies of Electrical Equipment for L. and Y.R., C.L.R., and G.N.P. and B.R., from Chapters V., VI. and VII.

chapter. But for these two points the motors are taken at the rating of 200 hp for which they are sold. It is, however, well known that these motors rate at 240 hp when referred to the 1-hour, 75° C. basis. The data given in Table XXXV. shows that on this latter rating, the average loads in the two cases were 41·6 per cent. and 47·5 per cent. of the (1-hour, 75° C.) rated load.

The two results indicated by the two crosses in Fig. 60 represent the values in the last column of Table XXXV. The L. and Y. results of Chapter V. are indicated by the two triangles, and the C.L.R. results of Chapter VI. are indicated by the circles, the double circle representing the normal 6-coach train.

TABLE XXXV.—SHOWING THE LOADING OF THE GREAT NORTHERN PICCADILLY AND BROMPTON RAILWAY'S MOTORS DURING THE TESTS A AND B OF TABLE XXXIII.

Average load in hp while electricity is on.—A.	Over-all efficiency of equipment in actual service.	Percentage which A is of rated load when the rating is taken as—	
		200 hp.	240 hp.
100	65·5	50·0	41·6
114	69·0	57·0	47·5

We thus see that, far from being contradictory, the results obtained on these different tests on three different railways, are in remarkable agreement, considering the inevitably rough practical nature of such tests on the road.

We may, with considerable assurance, take the curve drawn in Fig. 60 amongst these points, to be fairly representative of modern equipments of series-wound, continuous-electricity railway motors. This curve teaches us the important lesson that there is a better reason than saving in weight, for advocating the application of forced draught to continuous equipments, since we could operate smaller motors at a much higher average load, and with a considerable improvement in the over-all efficiency of the equipment. With the excellent commutation attainable with modern interpole motors, the plan should be very practicable, since, even at the high overloads encountered during the accelerating period, excellent commutation may be relied upon.

Mr. J. R. Chapman, Chief Engineer of the Underground Electric Railways Co. of London, has supplied the author with some valuable data bearing on the question of the most economical schedule speed. The data applies to the Great Northern Piccadilly and Brompton Railway. From March 11 to April 29, 1907, the trains on this railway were operated to a schedule speed of 14·82 ml ph. From April 29 to October 14, 1907, the schedule speed was 15·24 ml ph, and after October 14, 1907, the schedule speed was 16·1 ml ph.

The following results are not in the form in which they were supplied to me, but are so arranged as to better supplement the earlier data in this chapter. The calculations involve certain assumptions which must, however, be closely correct.

Designation of service.	I.	II.	III.
Schedule speed (ml ph).	14·8	15·2	16·1
Single-trip time (minutes)	36	35	33
Average number of coaches per train . .	3·30	3·17	4·00
Estimated average weight of train, with passengers (tons)	67	65	80
Number of motor-coaches per train . .	1	1	1
Consumption per ton-mile (from data supplied) (w hr)	67·0	78·0	88·7
Train-miles per week	39,000	39,000	42,500
Consumption per train-mile (kw hr) . .	4·50	5·06	7·10
Consumption per week (kw hr) . . .	176,000	198,000	302,000
Consumption reduced to reference basis of 40,000 train-miles per week . . .	180,000	203,000	284,000
Further reduced to reference basis of 70-ton trains	188,000	218,000	248,000
Cost of electricity per week (taken at 1*d.* per kw hr)	£785	£910	£1035

With the 16·1 ml ph service, two less trains are required to provide the aggregate of 40,000 train-miles per week, than are required for the 14·8 ml ph service. Thus, from the (£1035 − £785 =)£250 per week greater cost for electricity, is to be deducted the cost of working two trains and the capital charges associated with them. It should also be remembered that the faster service constitutes a feature calculated to attract an increased number of passengers. This leads to a greater percentage of seats occupied. Furthermore, it is better to work a train at high speed, realize its earnings, scrap it after it has done its reasonable mileage and buy a new train, than spare it to the extent of spreading out over a greater term of years its total earning capacity. The former plan will obviously yield a greater return for a given amount of capital invested in rolling stock.

CHAPTER VIII

ACCELERATION AND TRACTIVE FORCE

THE acceleration due to gravity is usually denoted by the letter g. When expressed in meters per second per second (m psps) we have—

$$g = 9{\cdot}81.$$

But 1 m psps is equal to 2·24 ml phps. Consequently, when accelerations are expressed in ml phps, we have, for the acceleration due to gravity—

$$g = 9{\cdot}81 \times 2{\cdot}24 = 22{\cdot}0.$$

We also have the relation—

$$F = \frac{W}{g} \times a$$

for the tractive force required to accelerate at the rate a, a body of weight W when moving in a frictionless medium. Consequently, if a, the acceleration, is expressed in ml phps, and if the body weighs 1 ton (*i.e.* if W = 1·00), then, for the tractive force in tons per ton, we have the expression—

$$F = \frac{a}{22{\cdot}0}.$$

For an acceleration of 1 ml phps, we have—

$$F = \frac{1}{22{\cdot}0} = 0{\cdot}0455 \text{ ton per ton of weight accelerated.}$$

$$= 45{\cdot}5 \text{ kg per ton of weight accelerated.}$$

While the entire train is being accelerated, producing a corresponding increase in its translational momentum, certain parts—the wheels, axles and motor armatures—are gathering rotational momentum, and will need a corresponding amount of energy to provide such momentum.

This point was considered in Chapter IV., where an additional 9 per cent. was added to the translational energy to give the total

energy (*i.e.* including the rotational component). The same allowance should be made here, and, therefore, when accelerating at 1 m phps, the figure of 45·5 kg per *ton of material* accelerated becomes 49·5 kg per *ton of train* accelerated.

Thus, we have the simple rule that when a train is being accelerated, the component of the total tractive force required to provide the momentum (*i.e.* exclusive of the component required to overcome train-friction) is, per ton weight of train, and per ml phps of acceleration, equal to 49·5 kg.

Thus, if a train weighs 70 tons, and if it is desired to bring it up to its crest speed, with an average acceleration of 0·90 ml phps, then the average tractive force during the accelerating period (exclusive of the amount required to overcome train-friction), is equal to—

$$49{\cdot}5 \times 70 \times 0{\cdot}90 = 3120 \text{ kg},$$

or 3·12 tons. If the average value of the train-friction during this period is 6 kg per ton, then the average tractive force required to overcome train-friction is equal to—

$$6 \times 70 = 420 \text{ kg},$$

or 0·42 ton. Consequently, in this case, the average value of the *total* tractive force is equal to—

$$3{\cdot}12 + 0{\cdot}42 = 3{\cdot}54 \text{ tons}.$$

The average tractive force per ton is made up of—

$$49{\cdot}5 \times 0{\cdot}90 = 44{\cdot}5 \text{ kg per ton for acceleration, and}$$
$$6{\cdot}0 \text{ kg per ton for friction,}$$

resulting in a value for the average tractive force of—

$$44{\cdot}5 + 6{\cdot}0 = 50{\cdot}5 \text{ kg per ton},$$

or a total tractive force of $(0{\cdot}0505 \times 70 =)3{\cdot}54$ tons for the 70-ton train.

In Fig. 61 is given a speed-time diagram for a run of 0·48 mile from start to stop. The run is accomplished in 95 seconds. Consequently, the average speed is—

$$\frac{3600}{95} \times 0{\cdot}48 = 18{\cdot}2 \text{ ml ph}.$$

After 48 seconds from the start the train has attained its crest speed of 25·5 ml ph. Consequently, the average acceleration during these 48 seconds is—

$$\frac{25{\cdot}5}{48} = 0{\cdot}53 \text{ ml phps}.$$

The acceleration is, however, varying during these 48 seconds.

At any given instant it is equal to the rate of change of speed at that instant. Taken over any very short interval of time, the curve is approximately straight. Thus, over the short portion extending from a speed of 14 ml ph to a speed of 16 ml ph, we may take it as a straight line. By drawing the small triangle indicated in Fig. 61 we see that the change of 2 ml ph in the speed (*i.e.* the change of from 14 to 16 ml ph) occupies 2·7 seconds, for the speed is seen to be 14 ml ph 14·8 seconds after the start, and it is 16 ml ph after 17·5

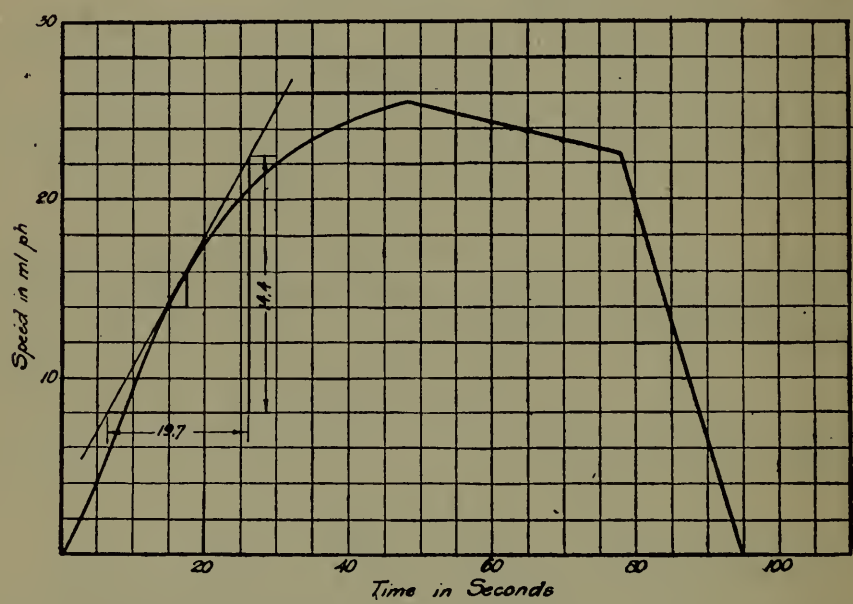

FIG. 61.—Determination of Acceleration and Power from Speed-time Diagram.

seconds from the start. The average acceleration corresponding to the mean speed of—

$$\frac{14 + 16}{2} = 15 \text{ ml ph}$$

is consequently—

$$\frac{16 - 14}{17{\cdot}5 - 14{\cdot}8} = \frac{2{\cdot}0}{2{\cdot}7} = 0{\cdot}74 \text{ ml phps.}$$

If we extend the hypotenuse of this little triangle, we obtain a tangent to the curve at the point corresponding to a speed of 15 ml ph and to—

$$\frac{14{\cdot}8 + 17{\cdot}5}{2} = 16{\cdot}2 \text{ seconds}$$

from the start. By measuring the slope of this tangent in the scale of the axes, we obtain (see the large triangle)—

$$\frac{22{\cdot}4 - 8{\cdot}0}{26{\cdot}2 - 6{\cdot}5} = \frac{14{\cdot}4}{19{\cdot}7} = 0{\cdot}73 \text{ ml phps.}$$

(The base of this triangle may be conveniently arranged to measure an even figure—say 20 in this case—the result being, of course, the same, $\frac{21{\cdot}6 - 7{\cdot}0}{20} = 0{\cdot}73$.)

From the construction it is obvious that the result is necessarily the same as that obtained by the first calculation for the small triangle. Owing to the greater length of the hypotenuse, and the greater accuracy thereby obtained in the readings, the method of determining the acceleration at any point of the curve by drawing a tangent to the curve, and passing through that point, is the most useful and accurate graphical construction by means of which to calculate the acceleration.

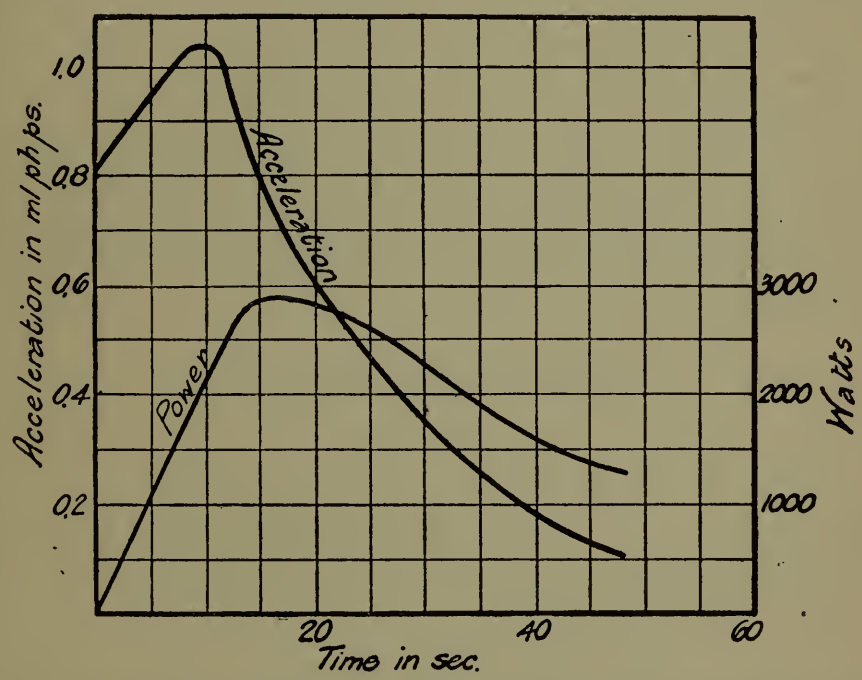

FIG. 62.—Acceleration and Power Curves deduced from the Speed-time Diagram in Fig. 61.

Otherwise expressed, we may say that the acceleration corresponding to any point of the speed-time diagram is proportional to the slope of the tangent to the curve at that point (the slope being the ratio of the vertical and horizontal distances between two points on the tangent, both distances being measured to the scales of the corresponding axes). Thus, the steeper the curve at any point, the greater the corresponding acceleration or deceleration, as the case may be.

In Fig. 62 the upper curve represents the values of the acceleration for the first 48 seconds of the diagram in Fig. 61.

It has been obtained by drawing lines tangent to the speed-time

diagram at each successive 5 seconds from the start. The values of the slopes (*i.e.* the values of the ratio of the increment in speed to the corresponding increment in time) are equal to the values of the corresponding acceleration. Thus, referring again to Fig. 61, we see that the speed of 15 ml ph, for which we obtained 0·73 for the value of the corresponding acceleration, occurred 16·2 seconds after the instant of starting from rest, and, turning to the upper curve in Fig. 62, we find that, corresponding to an abscissa of 16·2 seconds, the value plotted for the acceleration is 0·73 ml phps. Furthermore, if we obtain the mean ordinate of this acceleration curve for the 48 seconds which elapse from the start up to the crest speed of 25·5 ml ph, we find it to be 0·53 ml phps, which, as we have already seen, is equal to the crest speed (25·5 ml ph) divided by the time (48 seconds) elapsing while this speed was being acquired. From this acceleration curve we could construct a curve showing the corresponding values of the tractive force per ton, by the rule that an acceleration of 1 ml phps requires (neglecting friction) a tractive force of 49·5 kg per ton, and by adding a further 6 kg per ton for the tractive force required to overcome friction. This portion of the calculation can, however, be well worked out tabularly, as in Table XXXVI.—

TABLE XXXVI.—CALCULATIONS OF THE POWER CURVE FOR THE SPEED-TIME DIAGRAM INDICATED IN FIG. 61.

Interval (seconds).	Mean accelerating rate during interval (ml phps).	Tractive force (kg per ton).		Total tractive force (kg per ton).	Mean speed during interval (ml ph).	Mean speed in meters per sec. (0·447 × speed in ml ph).	Power per ton (kg m per sec.).	Power per ton (watts).
		For acceleration.	For friction.					
0–5	0·90	44·5	6·0	50·5	2·6	1·16	59	580
5–10	0·99	49·0	6·0	55·0	6·8	3·04	167	1640
10–15	0·93	46·0	6·0	52·0	12·0	5·37	279	2740
15–20	0·70	34·6	6·0	40·6	16·0	7·15	290	2840
20–25	0·54	26·7	6·0	32·7	19·0	8·50	278	2730
25–30	0·41	20·3	6·0	26·3	21·2	9·50	250	2450
30–35	0·30	14·8	6·0	20·8	22·8	10·20	212	2080
35–40	0·22	10·9	6·0	16·9	24·0	10·73	181	1780
40–45	0·15	7·4	6·0	13·4	24·7	11·05	148	1450
45–48	0·12	6·0	6·0	12·0	25·4	11·35	136	1330

By multiplying the total tractive force (kg) at any instant by the speed in m ps at that instant, we obtain values of the power in kg m ps. Now, since—

$$1 \text{ kg m ps} = 9{\cdot}81 \text{ watts},$$

we can obtain the power in watts by multiplying the above values for kg by 9·81.

The figures in the last column, which are plotted in the lower curve in Fig. 62, represent the power delivered by the motors to the axles of the train. The average value is 1960 watts, while the maximum value is 2840 watts, or 45 per cent. greater than the average. These figures relate to the 48 seconds during which the train is drawing electricity from the line. The time from *start* to *start* is, however—

$$95 + 15 = 110 \text{ seconds.}$$

Consequently, the average power required at the axles over the whole run is—

$$\frac{48}{110} \times 1960 = 855 \text{ watts per ton.}$$

The ratio of the maximum value to this average value for the entire run is 3·3.

All these figures have related to the **output from** the electrical equipment. To change from power to energy, let us write down the 855 watts per ton as 855 w hr per ton per hour. Now, since the schedule speed is 15·7 ml ph,* the average output of the motors is—

$$\frac{855}{15{\cdot}7} = 54{\cdot}5 \text{ w hr per ton-mile.}$$

Working this out by our earlier methods of adding together the energy required for momentum and friction, we arrived at the result of 53·5 w hr per ton-mile (see Chapter VI. p. 90). The discrepancy (of 2·0 per cent.) is very slight, and is due to taking averages of ordinates separated from one another by so great an interval as 5 seconds, and also to errors in deriving by graphical methods the acceleration curve from the speed-time diagram.

It may, in general, be said that the subject is best approached from the standpoint discussed in the previous chapters, when the object is to estimate the input in w hr per ton-mile, and from this acceleration-curve standpoint, when studying the *instantaneous* values of the power required.

In general practice it will be found that trains are equipped with sufficient motor capacity to ensure that the average load, *taken over the entire journey*, shall be only some 20 to 25 per cent. of the rated load on the 1-hour, 75° basis of rating. Taking such a case as that which we have just investigated, and assuming that it is desired that

* The speed-time diagram of Fig. 61 is reproduced from Fig. 51 of Chapter VI., where it will be found that the duration of stops is 15 seconds and the schedule speed 15·7 ml ph.

the motors' average load shall be only 25 per cent. of their rated load, then we find that the motors will, once during each run from start to stop, and consequently $\left(\frac{15 \cdot 7}{0 \cdot 48} = \right)32 \cdot 8$ times per hour, sustain a temporary load of—

$$3 \cdot 3 \times 25 = 82 \cdot 5 \text{ per cent. of their rated load.}$$

The ratio of maximum to average load, taken over the entire journey, is often of the order of from 4 to 5 instead of the relatively lower ratio of 3·3 obtained in this example.

For services where such high maxima would be occurring, it has been considered desirable to keep the average load down to from 20 per cent. to 25 per cent. of the rated load. Thus, in a case where the average load is 20 per cent. of the rated load, and the maximum load is 5 times the average load, then the maximum load would work out at $5 \times 20 = 100$ per cent. of the rated load. In most instances of approved modern practice, the maximum load to which the motor is normally subjected at frequently recurring intervals has rarely been greater than the 1-hour, 75° C. rated load. Of course, for occasional grades, such as might occur at points in the route, higher loads are sustained, but for the maximum load periodically occurring during each run from start to stop, the practical experience of years has led to setting this limit. When a motor-coach is brought in at the end of a day's constant running at its normal schedule, *i.e.* after from 15 to 18 hours, the thermometric temperature rise of the motors should preferably not be more than 65° above the temperature of the surrounding air. On the basis of this limitation, the maximum temperature at the interior of the windings will usually be at least 80° above the surrounding air. If the surrounding air should be at a temperature of 25°, this would bring the actual temperature of the hottest parts of the motor to at least 105°.

Examples.

1. Calculate the average tractive force required on a straight and level track to accelerate from rest up to a speed of 10 ml ph, in 17 seconds, a train weighing 200 tons. (Allow 9 per cent. for the momentum of the rotating parts, and take the train-friction at 6 kg per ton.) *Ans.* 7000 kg.

2. For the conditions in question 1, at the moment when the speed of 10 ml ph has been reached, what is the power being delivered from the motors to the driving axles? *Ans.* 307 kw.

3. What would have to be the percentage down grade in order that the above train, after acquiring a speed of 10 ml ph, should continue running at that speed without being supplied with power? (Train-friction, 6 kg per ton.) *Ans.* 0·6 per cent.

4. Retaining the same tractive effort as in questions 1 and 2, how long would it take the train to acquire a speed of 10 ml ph if, instead of starting on a level, it started on an up-grade of 1 per cent.? *Ans.* 26 seconds.

5. If, on the other hand, it is desired that it shall still acquire the speed of 10 ml ph in 10 sec., notwithstanding the 1 per cent. up-grade, (*a*) what tractive effort must be provided, and (*b*) what will be the power delivered to the axles at the instant the speed reaches 10 ml ph? (Train-friction, 6 kg per ton.)
Ans. 13,100 kg; 575 kw.

6. If the 200-ton train starts on a down grade of 2 per cent., what tractive effort must be supplied to provide a constant acceleration of 0·9 ml phps. (Train friction, 6 kg per ton.) *Ans.* 6100 kg.

CHAPTER IX

TRAIN-FRICTION

It has already been explained that the advantage of electricity over steam is most marked for services where the trains stop frequently. Since frequently-stopping trains are seldom or never running at constant speed for more than a fraction of a minute at a time, the energy required to supply the momentum corresponding to the crest speed usually constitutes, as we have seen, the greater part, or, at any rate, a very large part, of the total energy consumed by the train, and the energy required for overcoming train friction is but a small proportion of the total energy consumed. Let us now turn our attention to the other extreme, where the trains stop only at rare intervals, and where, consequently, the energy required to supply the momentum is but an insignificant portion of the total energy consumed. If we neglect this portion, then there remain but two components. One of these components is the energy required for overcoming train-friction, *i.e.* the energy transformed into heat at the bearings, at the contacts of the wheels with the rails, and in air friction, and the other is the energy supplying the losses in the electrical equipment, *i.e.* the energy transformed into heat in the electrical equipment. If the electrical equipment of the train were of 100 per cent. efficiency, then the total energy consumed would be exclusively that required for train-friction.

For certain reasons which I shall endeavour to explain in the course of this chapter, it is very difficult to analyse the tests which have been made to determine train-friction under various circumstances. It appears, however, conclusive that when, over a level and well-built permanent way, a train is running at a constant speed, the train-friction is very much less than when the train is driven at varying speeds, the mean value of which is numerically equal to this constant speed. Whereas for speeds fluctuating between 0 and 30 or 40 ml ph we have taken 6 kg per ton as a suitable basis for estimating the friction component, the appropriate values for *constant* speeds throughout this range, are much less. Several considerations influence the values appropriate for different cases, but for a

representative 100-ton passenger train, the values in Table XXXVII. are illustrative of the dependence of the tractive force on the speed.

TABLE XXXVII.—TRACTIVE FORCE REQUIRED FOR THE PROPULSION AT CONSTANT SPEED OF A 100-TON TRAIN.

Speed (ml ph).	Tractive force required to overcome train resistance (kg per ton).
10	1·5
20	2·5
30	3·6
40	4·7
50	6·6
60	8·3
80	12·8
100	18·5

The last few speeds in the above table are, of course, much higher than can be employed with frequently stopping trains, but they are given as of general interest. It is often desirable to add 15 per cent. to the values in the above table in order to be conservative.

Let us take the case of a 100-ton train running at a constant speed of 30 ml ph. We see from the above table that the tractive force would be of the order of $3{\cdot}6 \times 1{\cdot}15 = 4{\cdot}15$ kg per ton. Since there are 1609 meters in one mile, and since 1 w hr is equal to 367 kg m, the energy required at the axles to overcome train-friction is, in this case, of the order of—

$$\frac{4{\cdot}15 \times 1609}{367} = 18{\cdot}1 \text{ w hr per ton-mile.}$$

If the train is propelled by motors so geared that, when running at their rated speed, the train travels at 30 ml ph, then the conditions for this constant-speed equipment will be so very much more favourable than in the case of the series-parallel equipments employed for frequently stopping trains, that it will be conservative to take the over-all efficiency of the electrical equipment as high as 80 per cent. The input to the train will then be—

$$\frac{18{\cdot}1}{0{\cdot}80} = 22{\cdot}6 \text{ w hr per ton-mile.}$$

Since the train's speed is 30 ml ph the input may also be expressed as $(22{\cdot}6 \times 30 =)$ 677 w hr per ton per hour, or we may say that the *power* required by the train is 677 watts per ton. The total power required for the 100-ton train is 67,700 watts. The output from the motors is—

$$\frac{67{,}700 \times 0{\cdot}80}{746} = 72{\cdot}5 \text{ hp.}$$

Two 40-hp motors would be ample for the work. For such an equipment we should not employ the 1-hour rating; for since the train runs at constant speed, the motors would be designed to carry their rated load continuously without undue temperature rise. The motors would be of the totally enclosed type. The weight of such motors is less the greater the rated speed in rpm, but even if they are designed for low speed, the weight of the total electrical equipment would only be of the order of 5 tons. With forced circulation of air the weight of the electrical equipment could be still further decreased, but there would not be sufficient justification for this, since in any event, the weight of the electrical equipment is only some 5 per cent. of the total train weight.

A 100-ton train for this same schedule speed of 30 ml ph but designed and equipped to make one 20-second stop per mile, would have an energy consumption of some 160 w hr per ton-mile (see Fig 45), and would have to carry an electrical equipment (on the 1-hour, 75° C. basis of rating) of 18 hp per ton, or a total of 1800 hp. If not ventilated by forced draught, such a train would probably carry an electrical equipment weighing at least 15·5 kg per rated hp. The total weight of electrical equipment is thus—

$$0{\cdot}0155 \times 1800 = 28 \text{ tons.}$$

Of the total train weight of 100 tons, some 28 per cent. would be represented by the electrical equipment. The weight of the electrical equipment of this stopping train is at least some five times as great as that of the constant-speed train for the same schedule speed.

This rough comparison has been traced through for the purposes of again emphasising the predominating influence of the momentum in the operation of city and suburban passenger trains.

The data which engineers usually have in mind as regards the coal burned on a locomotive per train-mile are based on runs of very considerable distances between stops, and the natural mistake is liable to be made of comparing these coal consumptions per train-mile with the coal consumption figures obtained at Electricity Supply Stations from which electric trains obtain their power. Obviously, it must be remembered that electric trains are usually operated on services with frequent stops, and require at the train much greater amounts of power than are required by trains hauled by steam locomotives. Thus, at 50 ml ph, a non-stopping 100-ton electric train only requires some—

$$\frac{6{\cdot}6 \times 1{\cdot}15 \times 1609}{367 \times 0{\cdot}80} = 42 \text{ w hr per ton-mile.}$$

This is only half as much energy per ton-mile as is required by a 16 ml ph 100-ton train making 2 stops per mile. The point to be noted

is that the particular field for which electric propulsion is so admirably appropriate is in operating city and suburban services, and that if steam locomotives could provide the high speeds, together with the frequent stops, which are attainable by electrical methods, the coal consumption of such steam trains would be much in excess of the coal consumption of much faster steam trains making only infrequent stops.

The values given in Table XXXVII. only relate to the train-friction, and are exclusive of the friction of the gearing through which the power is transmitted from the electric motors to the axles. The motor gearing is in most calculations conveniently considered as part of the motor, and the gear-friction loss is considered as one component of the total loss in the motor. But there is the difference that, whereas all the other losses in the motor cease at the instant when the electricity is cut off from the train, the *gear loss* continues so long as the train is in motion. Thus, the deceleration during coasting (*i.e.* drifting) is proportional to the train-friction plus the gearing friction. The gearing friction is by no means negligible in comparison with the train-friction. On the contrary, it constitutes a very important component of what we may, for convenience, term the "inclusive" or "over-all," friction.

The question is so important as to justify us in working out a case. Let us consider a motor-coach with an electrical equipment comprising two 150-hp motors. The complete weight of the motor-coach, including the electrical equipment, is 39 tons. At a speed of 45 ml ph the frictional resistance, exclusive of gearing, may be taken at 5·4 kg per ton, this value corresponding roughly with the data in Table XXXVII. If the load consisted exclusively of the train friction, then at 45 ml ph the output from the two motors would be calculated as follows:—

$$\text{Total tractive force} = 39 \times 5{\cdot}4 = 210 \text{ kg}$$

$$\text{Speed} = \frac{45 \times 1609}{3600} = 20{\cdot}1 \text{ m ps}$$

$$\begin{aligned}\text{Output} &= 210 \times 20{\cdot}1 = 4230 \text{ kg m ps}\\ &= 4230 \times 9{\cdot}81 = 41{,}400 \text{ watts}\\ &= \frac{41{,}400}{746} = 55{\cdot}5 \text{ hp}\end{aligned}$$

This is, of course, a very small load for an equipment with 300 aggregate rated hp; the output per motor is only—

$$\frac{55{\cdot}5}{2} = 27{\cdot}6 \text{ hp},$$

or 18·4 per cent. of the rated load of the motor.

At rated load of 150 hp the gear loss in this motor is 4 per cent. of the output, thus being—

$$150 \times 746 \times 0{\cdot}04 = 4480 \text{ watts.}$$

The gearing loss in railway motors may, for rough calculations, be taken constant at all loads. At 45 ml ph the friction loss of the train, inclusive of gearing, is $41{,}400 + 2 \times 4480 = 50{,}360$ watts, and this is the load delivered from the armature axles of the motors to their gearing. The train-friction, *inclusive of gearing*, is thus—

$$\frac{50{,}360}{41{,}400} \times 5{\cdot}4 = 6{\cdot}6 \text{ kg per ton,}$$

of which $(6{\cdot}6 - 5{\cdot}4 =)1{\cdot}2$ kg per ton is due to the gearing. Had the motor-coach been provided with four motors instead of with two motors (and this is very often the case), then the friction of the motor-coach would have been $(5{\cdot}4 + 2 \times 1{\cdot}2 =)7{\cdot}8$ kg per ton. In this case we should have a $\left(\frac{7{\cdot}8}{5{\cdot}4} = 1{\cdot}45\right)45$ per cent. higher frictional resistance per ton than would be obtained with a trailer-coach. In this case, when all the four axles carry motors, the increased electrical equipment will bring the weight of the motor-coach up to 46 tons. Let us, from this point onwards, keep such a four-motor equipment in mind. The equipment is amply able to operate a heavy train of trailers at the speed in question, namely, 45 ml ph. Let us add five trailers, each of a weight of 26 tons. Thus, the five trailers will, at 45 ml ph have a resistance of $(5 \times 26 \times 5{\cdot}4 =)700$ kg as against the resistance of $(46 \times 7{\cdot}8 =)360$ kg for the four-motored coach which hauls the train. The total weight of the train is—

$$46 + (5 \times 26) = 176 \text{ tons,}$$

and the *average* train-friction is—

$$\frac{700 + 360}{176} = \frac{1060}{176} = 6{\cdot}0 \text{ kg per ton.}$$

The longer we make the train by the addition of trailers, the more nearly will the train-friction approach 5·4 kg per ton. In Table XXXVIII. are worked out the values for trains with one motor-coach, and successively 1, 2, 3, 4 and 5 trailers.

TABLE XXXVIII.—SHOWING THE INFLUENCE ON THE TOTAL FRICTIONAL RESISTANCE OF A TRAIN, OF ADDING TRAILER-COACHES.

Number of coaches in train.	Make-up of train: M = motor-coach. T = trailer-coach.	Weight of train (tons).	Weight of motor-coach component (tons).	Frictional resistance of motor-coach component (kg), *i.e.* 7·8 × weight.	Weight of trailer-coach component (tons).	Frictional resistance of trailer-coach component (kg), *i.e.* 5·4 × weight.	Total frictional resistance of train (kg).	Frictional resistance of train (kg per ton).
1	1M	46	46	360	—	—	360	7·8
2	1M + 1T	72			26	140	500	7·0
3	1M + 2T	98			52	280	640	6·5
4	1M + 3T	124			78	420	780	6·3
5	1M + 4T	150			104	562	922	6·1
6	1M + 5T	176			130	700	1060	6·0

In the above example I have laid stress on one of the two important influences in virtue of which long trains (usually also *heavy* trains) demand a lower tractive force per ton of weight than short (usually also *light*) trains.

A second influence the importance of which is very dependent upon the speed, relates to air friction. Air friction only commences to be of importance at speeds of some 30 ml ph, and the air resistance in kg per ton is less the greater the length of the train. The train-friction values in Table XXXVII. were given for 100-ton trains, so that they will be applicable to the 3-coach train of Table XXXVIII. Now, with a shorter train, the frictional resistance per ton, other than air friction, will increase in the manner which has just been determined, but the air resistance per ton will also be greater, due to the predominating influence of the air friction at the front and rear of the train, and will tend to accentuate the differences. Considering, then, the resistance of the 3-coach train as correct, the values for the *other* trains, when this change of air resistance is duly considered, will conform more to the dotted curve in Fig. 63, where the full-line curve is that plotted from the results in Table XXXVIII. The frictional resistance of the 3-coach train is taken the same in both cases.

A series of tests were made at Zossen,* near Berlin, in 1903, to determine the frictional resistance of electric trains at high speeds. Some of the results are given in Table XXXIX. (p. 122), where the component resistances are shown. It is seen from the table that at about 40 ml ph the air resistance, for the conditions of the Zossen

* A detailed description of the Berlin-Zossen Tests is given in "Electric Railway Engineering," by Parshall and Hobart (Constable and Co., 1907).

tests equalled the remainder of the frictional resistance. In the values of the frictional resistance given in Table XXXVII. the air resistance has been included as part of the total train resistance.

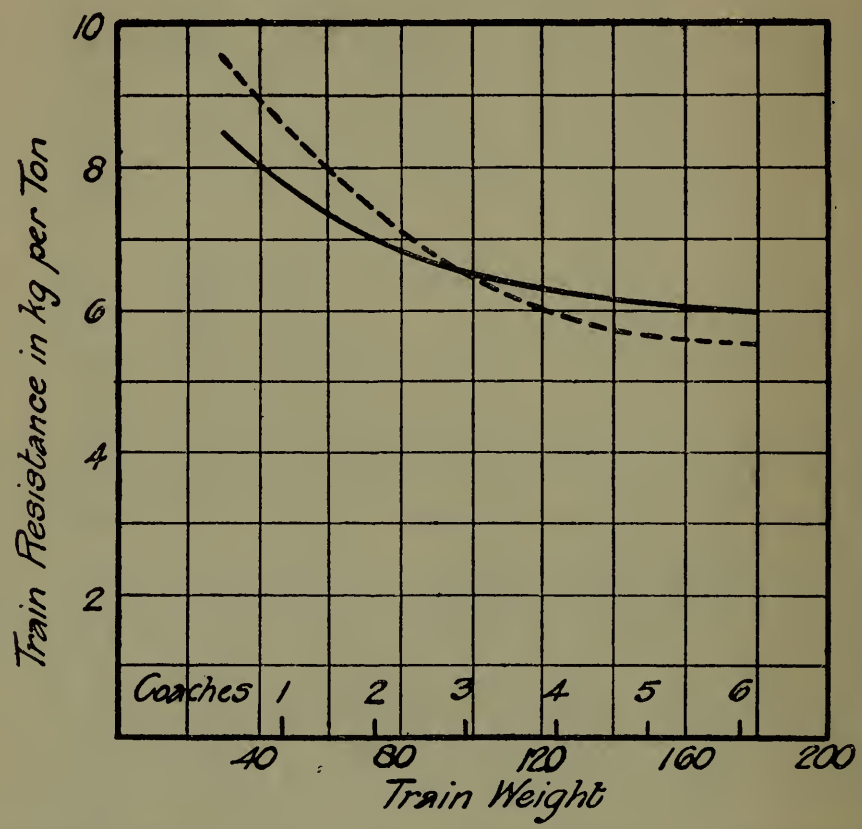

Fig. 63.—Curves showing Tendency of Train Resistance to decrease with Increased Weight of Train.

Table XXXIX.—Values of Frictional Resistance deduced from the Berlin-Zossen Tests with an 83-ton, 75-foot Coach.

Speed in ml ph.	Mechanical resistance (track and axle friction) (kg per ton).	Air resistance (kg per ton)	Total frictional resistance (kg per ton).
40	2·2	2·3	4·5
50	2·4	3·6	6·0
60	2·6	5·3	7·9
70	2·8	7·0	9·8
80	3·0	9·0	12·0
90	3·2	11·4	14·6
100	3·4	14·2	17·6
110	3·6	17·8	21·4

A third factor, which does not in reality affect the train resistance, but which at first sight obscures the matter, is the varying efficiency

of the electrical equipment with varying load. Time and again, tests have been published in various countries, the results of which appeared to indicate a very remarkable diminution in the train resistance per ton, for long trains, as compared with short trains. I am of opinion that much too great a portion of the decreased input to the train (per ton of its weight) with increased weight of train, has been ascribed to decreased air friction, and that a factor of great importance has been overlooked. This factor relates to the increasing efficiency of the electrical equipment with increasing load, *i.e.* with increasing number of trailers hauled. The curve in Fig. 60, on p. 105, shows conclusively that there is a very considerable increase in efficiency with increasing load, even for a comparatively small range in the values of the loads carried by the motors. The results in Fig. 60, however, refer to runs with frequent stops, and are therefore not directly applicable to the present case of constant-speed running. The variation in the efficiency of the electrical equipment is even more pronounced when a test is first run with a single motor-coach at such a speed, or under such conditions, that the equipment is only very lightly loaded, corresponding tests afterwards being made with the motor-coach hauling enough trailers to load the motors well up toward their rated capacity.

Let us take our 46-ton motor-coach, with its equipment of four 150-hp motors, and let us roughly estimate the input for a speed of 45 ml ph, first when the motor-coach is running without any trailers, and afterwards with 2, 4, 6 and 8 trailers. The motor-coach weighs 46 tons, and each trailer-coach weighs 26 tons. Consequently, our calculations will relate to 5 trains weighing respectively 46, 98, 150, 202 and 254 tons. Let us ignore the variations in the air resistance per ton with increased length of train, in order to be the better able to study the influence to which we wish at this point to chiefly direct our attention. The calculations in this case are facilitated by considering the gearing to be part of the motor. We shall consequently take the train-friction throughout at 5·4 kg per ton. The tractive forces in the five cases are thus, 248, 529, 810, 1090 and 1370 kg, as set forth in Table XL. From the tractive force we determine the hp output of the motors when operating the trains at 45 ml ph (*i.e.* 20·1 m ps) in the same way already employed earlier in this chapter. There being four motors, the output per motor is 25 per cent. of the total output; and then by referring to the efficiency curve of a typical motor of the same rated hp (Fig. 43 on p. 72) the respective efficiencies of the motor at these outputs are obtained. It is seen that the first train, consisting of a motor-coach and no trailers, only requires 11 per cent. of the rated output of the motors when running at this constant speed, and the efficiency of the motors consequently has the low value of 40 per cent. During these tests the motor efficiency is the only

factor that need be considered, in order to arrive at the results in view. When two trailers are added, it is seen that the motor efficiency rises to 75 per cent., and that the load is 23 per cent. of the rated output. This case (the 98-ton, 3-coach train) is taken as the representative one for operation by one motor-coach, and the other trains are referred to it. In tests similar to those under consideration in this chapter, it would appear that the tractive force has frequently been erroneously deduced from the readings of the electrical input to the motors, and serious error would appear to have been involved, owing to taking a constant efficiency for the electrical equipment, *i.e.* it would appear that the output has been taken as being a constant ratio to the input, or that insufficient allowance has been made for the change in efficiency with load. For instance, in the case of the 5-coach train, the *true* motor efficiency is some 83 per cent., but if the 75 per cent. efficiency were adhered to (as in the standard 3-coach train) the *true* tractive force would be greater than the *apparent* tractive force (obtained by employing this 75 per cent. efficiency) in the ratio of—

$$(810) \text{ to } \left(810 \times \frac{75}{83}\right)$$

that is, in the ratio of 810 to 730. In other words, the equipment would have been credited with 8 per cent. less efficiency than it was really yielding, and hence the frictional resistance was in reality greater than was ascribed to it.

TABLE XL.—SHOWING THE INFLUENCE OF THE INCREASED EFFICIENCY OF ELECTRICAL EQUIPMENT (WITH INCREASED LOAD) ON THE FRICTIONAL RESISTANCE, AS DEDUCED FROM READINGS OF ELECTRICAL INSTRUMENTS DURING TESTS WITH AN ASSUMED CONSTANT EFFICIENCY FOR ALL THE TESTS.

Number of coaches in train.	Make-up of train: M = motor-coach. T = trailer-coach.	Weight of train (tons).	Frictional resistance (kg per ton) (gear loss included in motor losses).	True Frictional resistance (kg)	Aggregate output from all four motors (hp) at 45 ml ph (20 1 m ps).	Output per motor (hp).	Percentage which this output is of the rated load of motor (150 hp).	Efficiency of motor at this output, including gear loss (Fig. 43).	Apparent frictional resistance which would be obtained from electrical readings, assuming 75 per cent. efficiency throughout.	Corresponding apparent frictional resistance (kg per ton).	True frictional resistance, including gear friction (kg per ton).
1	1M	46	5·4	248	65·5	16·4	11	40	465	10·1	7·8
3	1M + 2T	98		529	140	35·0	23	75	529	5·4	6·5
5	1M + 4T	150		810	214	53·5	36	83	730	4·9	6·1
7	1M + 6T	202		1090	288	72·0	48	87	940	4·7	5·8
9	1M + 8T	254		1370	362	90·5	60	88	1170	4·6	5·6

A total tractive force of 730 kg corresponds to an apparent frictional resistance of 4·9 kg per ton, as against the true value (exclusive of gear loss) of 5·4 kg per ton. The error is particularly

striking in the first case in the table, where the train consisted of a single motor-coach, and where the efficiency, owing to the light load, is very low, and where, consequently, the *apparent* value for the tractive force is much greater than the *true* value.

The last column shows that the effect of overlooking the change in motor efficiency would be to give an erroneous impression that the anticipated result had been obtained—that is, that there had been a great diminution in the frictional resistance with increased train length. The apparent results accentuate beyond its true value, the

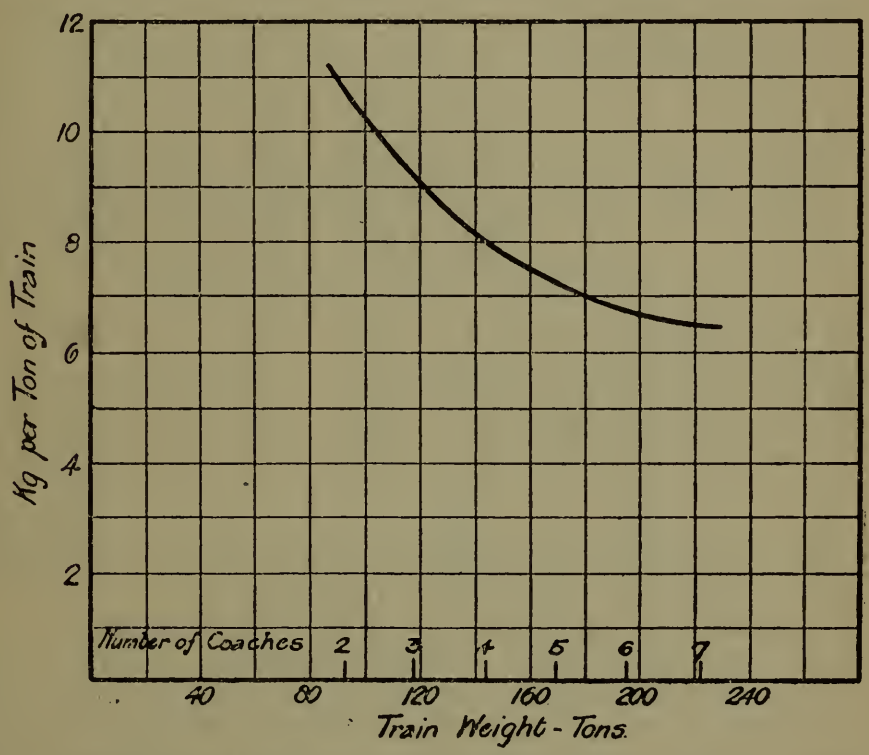

FIG. 64.—Curve showing Change of Tractive Resistance with Weight of Train as obtained from Results of Tests given in Table XLI.

diminution with increasing length of train, of the frictional resistance per ton weight of train.

Some interesting tests have been made by Armstrong, Potter, Aspinall and others to investigate the change of tractive resistance with increased train length, and the results fully confirm the general considerations which have been set forth in this chapter.

Aspinall has published results of some very complete tests* carried out with a view to obtain the effect of varying length of train. Table XLI. gives the average values obtained, and these are plotted in Fig. 64.

* "Proceedings of the Institution of Civil Engineers," vol. clxxix. p. 121.

Here it will be seen that the tractive force decreases from 10·7 kg per ton when the train consists of two motor-coaches and no trailers, down to 6·5 kg per ton when five trailers are added. If the trailers are only debited with a tractive force equal to the increase of the total input to the train over the input required by the motor-coaches alone the tests would appear to indicate that the tractive force per ton of added trailer would be of the values entered in the last column, *i.e.* some 3·5 kg per ton of trailer, as against some 10·7 kg per ton of motor-coach.

TABLE XLI.—AVERAGE VALUES OF THE RESULTS OBTAINED DURING TESTS OF THE TRACTIVE RESISTANCE ON LANCASHIRE AND YORKSHIRE RAILWAY.

Number of coaches in train.	Make-up of train: M = motor-coach. T = trailer-coach.	Length of train.		Weight of train (tons).	Speed of train observed 2½ minutes after start (ml ph).	Total tractive effort (kg) to overcome "over-all" friction, as calculated from readings.	Tractive effort for "over-all" friction, kg per ton of train.	Trailer weight added (tons).	Extra tractive effort (kg) to overcome friction of trailers.	Tractive force, kg per ton, for friction of trailers.
		(ft)	(m)							
2	2M only	124	38	92	51·7	984	10·7	—	—	—
3	2M + 1T	186	57	118	49·9	1071	9·1	26	87	3·4
4	2M + 2T	248	76	144	47·6	1148	8·0	52	164	3·2
5	2M + 3T	310	95	170	45·7	1230	7·3	78	246	3·2
6	2M + 4T	372	114	196	42·5	1320	6·7	104	336	3·3
7	2M + 5T	434	133	222	40·3	1458	6·5	130	474	3·6

While the published results are not accompanied by data permitting of confirming my opinion, it nevertheless appears to me that, notwithstanding that each motor-coach had four motors and, consequently, four sets of gearing, it is probable that, at the speeds indicated in the table, the tractive force, including the friction of the gearing, would, per ton of motor-coach, have been more of the order of 8 kg, and that the further 33 per cent. of apparent tractive force in the case of the 2-coach train is to be accounted for by the very high percentage of loss in the electrical equipment, since the motors, when no trailers were included in the train, were only loaded up to some 18 per cent. of their rated capacity, and would, consequently, have only very low efficiency. Conversely, a trailer friction of some 5 kg per ton in place of the 3·5 kg per ton could be arrived at by crediting the electrical equipment with a much higher efficiency, owing to the much greater percentage of rated load at which it was working. For it must be kept prominently in mind that a higher efficiency of the electrical equipment means a lower percentage of losses in the electrical equipment, and hence, for a given input, higher losses due to train resistance. These suggestions are only put forward as a view which, whether justified or not justified in the case of

these particular tests, should be carefully considered, as it has an important bearing on the subject of train-friction.

The higher tractive resistance of motor-coaches which we have considered, at once throws light on similar effects observed with electric locomotives in which geared motors are employed. Hutchinson * has carried out tests of the frictional resistance of some steam and electric locomotives.

The tests were first suggested by the energy required to haul trains through tunnels, when an idle steam locomotive constituted part of the trailing load. Hutchinson says, "The power required to haul these trains seemed greater than it should be; investigation showed that the difference was accounted for by the unexpectedly high frictional resistance of the steam locomotives as a trailing load." Tests were made (at a speed not stated in the paper) by towing the steam locomotives behind an electric locomotive fitted with test instruments, and the total tractive effort was deduced from these tests.

TABLE XLII.—VALUES OF THE FRICTIONAL RESISTANCE OF LOCOMOTIVES AS DETERMINED FROM TESTS BY HUTCHINSON.

Locomotive classification.	Total weight of locomotive with tender.	Net frictional resistance of locomotive and tender (kg).	Frictional Resistance (kg per ton).
Mallet No. 1904	250	4910	19·5
Mallet No. 1911	250	4090	16·3
Mallet No. 1905	250	7120	28·6
Consolidation.	150	2480	15·7
Pacific	188	1760	9·4
Electric	115	680	5·9

Previous tests on trailing coaches gave a figure of 2·7 kg per ton for the tractive resistance. This allowance was made for the electric locomotive in the above tests, and after making corrections for the grade, the frictional resistance was obtained. The results are given in Table XLII. for five steam locomotives and one electric locomotive. The frictional resistance of a steam locomotive and tender is seen to average 18 kg per ton—in one case reaching the high value of 28·6 kg per ton. The total resistance of the electric locomotive, including the friction of the gearing and motor bearings, was 5·9 kg per ton—a value over twice as great as that obtained for the trailing rolling stock.

Another set of test results of great interest, and bearing on the

* "Transactions of the American Institute of Electrical Engineers," vol. xxviii p. 1436.

subject of this chapter, are those obtained by Davis and analysed by Armstrong.* In Table XLIII. are given the results at which Armstrong arrived for the train-friction at various speeds for three cases—first a 280-ton train, then a train of 70 tons, and finally a 35-ton single coach. The tests indicated that trains heavier than 280 tons did not have an appreciably lower tractive resistance.

TABLE XLIII.—VALUES OF THE TRACTIVE RESISTANCE OF TRAINS OF VARIOUS WEIGHTS, AS OBTAINED FROM ARMSTRONG'S ANALYSIS OF TESTS BY DAVIS.

Speed (ml ph).	Tractive resistance (kg per ton).		
	280-ton train.	70-ton train.	35-ton single coach.
20	4·1	5·0	5·7
30	5·0	6·8	8·6
40	5·9	9·4	12·5
50	7·0	12·2	17·2
60	8·2	15·4	22·4
70	9·4	19·2	28·2
80	10·7	23·7	34·0

This chapter has so far been devoted to considerations of the tractive effort at constant speed; but there are similar effects of increased train weight on the energy consumption, when we come to services with frequent stops, such, for instance, as would occur in city and suburban services. As an instance of tests to determine the energy consumption for frequently-stopping trains of various weights, Table XLIV. is given. The table relates to tests made by Arnold and Potter.†

TABLE XLIV.—ARNOLD AND POTTER'S RESULTS OF TESTS FOR THE ENERGY CONSUMPTION OF TRAINS OF VARIOUS WEIGHTS UNDER SERVICE CONDITIONS (ONE MILE RUN).

Make-up of train: M = motor-coach. T = trailer-coach.	Weight of train (tons).	Weight of trailer-load (tons).	Maximum speed during the run (ml ph).	Average speed during the run (ml ph).	Energy consumption (from watt-meter) (w hr per ton-mile).
2M only	64·5	—	46·7	34·6	143·0
2M + 1T	85·0	20·5	44·7	33·1	127·0
2M + 2T	106·5	42·0	42·8	32·0	111·0
2M + 3T	133·5	69·0	41·0	30·6	104·0
2M + 4T	158·0	93·5	39·1	29·8	96·5
2M + 5T	181·0	116·5	37·9	28·6	91·1
2M + 6T	206·0	141·5	36·4	27·2	88·3

* "Transactions of the American Institute of Electrical Engineers," vol. xxii. p. 91.
† *Ibid.* vol. xix. p. 836.

The trains consisted first of two motor-coaches, and then successive additions of 1, 2, 3, 4, 5 and 6 trailer-coaches, each motor-coach weighing about 32 tons against an average trailer-coach weight of 23 tons.

Some interesting curves of train friction for varying weights of train have been plotted in Fig. 65. They are based on Aspinall's formula,* in accordance with which the train friction per ton of weight of train is equal to $a + \frac{V^{\frac{5}{3}}}{b + cL}$, where a, b and c are constants,

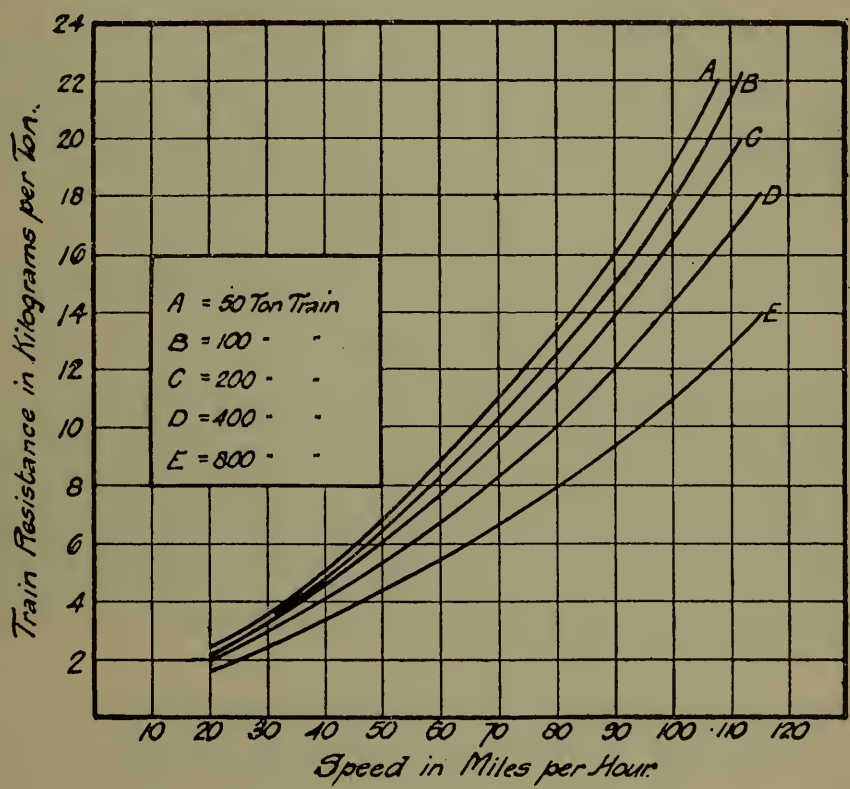

FIG. 65.—Curves showing the Tractive Resistance for Trains of Various Weights.

V is the speed of the train in ml ph, and L the length of the train in feet. By assuming a representative weight per foot of over-all length of train, the curves of Fig. 65 have been deduced for trains of a total weight of 50, 100, 200, 400 and 800 tons.†

Influence of Curves on Frictional Resistance

Curvature of the track adds very considerably to the tractive effort necessary for train propulsion, and the additional effort

* "Proceedings, Institution of Civil Engineers," vol. cxlvii. p. 241.

† The variation of the train friction, and also the variation of the energy consumption, with trains of different weights, have been investigated very fully at pp. 14–16 of "Electric Railway Engineering" (Parshall and Hobart).

required depends largely upon the degree of curvature, and also on the rigidity and gauge of the track, and on the wheel base of the coach.

The method by which track-curvature can be evaluated involves the angle which a given length subtends at the centre of the curvature. One way of expressing this curvature is by the degrees subtended by a chord 100 feet in length. The extra tractive effort required is taken as proportional to this figure. From numerous experiments to determine the train-friction on curves, an average of 0·32 kg per ton per degree has been found to agree closely with the various experimental results. This is the additional effort required on the track, quite apart from friction at the axles, ordinary friction between rail and wheel, and air friction. The most important formulae which have been put forward are those of Blondel-Dubois and Dupuy. The former gives the additional tractive effort as—

$$\frac{574}{R} \text{ kg per ton}$$

where R = Radius of curve in meters. The latter authority gives—

$$\frac{370}{R - 10} \text{ kg per ton.}$$

Both these formulae apply to the standard gauge of track.

Values for the additional tractive effort required on curves have been worked out by these three methods, and the results are set forth in Table XLV. It will be seen that they are in close agreement over the whole range of radii. The tractive resistance on any curve can then be computed from the values for straight track by adding the amount corresponding to the curve under consideration, as obtained from Table XLV.—

TABLE XLV.—ADDITIONAL TRACTIVE EFFORT REQUIRED ON CURVES.

Radius of curve.			Additional tractive effort required (kg per ton).		
Meters.	Feet.	Chains.	Estimated from the value of 0·32 kg per ton per degree.	Estimated from the formula of Blondel-Dubois.	Estimated from the formula of Dupuy.
30	99	1·5	19·10	19·20	18·50
40	132	2·0	14·10	14·30	12·30
50	165	2·5	10·90	11·50	9·26
60	198	3·0	9·10	9·57	7·40
80	264	4·0	7·05	7·19	5·29
100	330	5·0	5·45	5·75	4·11
120	396	6·0	4·55	4·79	3·36
160	528	8·0	3·45	3·59	2·46
200	660	10·0	2·77	2·87	1·95
300	990	15·0	1·81	1·92	1·27
400	1320	20·0	1·41	1·43	0·95
600	1980	30·0	0·91	0·96	0·63
1000	3300	50·0	0·55	0·58	0·37

Examples.

1. A 135-ton train maintains a schedule speed of 22 ml ph, making a 20-second stop every 0·9 ml. It consists of two motor-coaches and one trailer. Each motor-coach weighs 50 tons, and the trailer weighs 35 tons. Each of the four axles of each motor-coach carries a geared motor. On a straight and level, well-built track what would be your estimate of the average frictional resistance of this train when operating to the above schedule?

2. Allowing 12 per cent. for rotational momentum, and taking the over-all efficiency of the electrical equipment as 70 per cent. when operating to the above schedule, work out the energy consumption in w hr per ton-mile.

3. If, in the above case, 40 per cent. of the distance consists of curves of 150 meters radius, the remaining 60 per cent. being straight, estimate the energy consumption on the basis of the same over-all efficiency of the electrical equipment.

4. If, of the 135 tons total weight of train 40 tons represents the aggregate weight of the single-phase electrical equipment (which has a rated capacity of 920 hp), then, if 920-hp of continuous equipment weighing only 19 kg per hp, and comprising only 4 motors (instead of 8), were substituted, calculate the reduction in the input to the train, taking the rotational momentum at only 8 per cent. of the translational momentum, and again taking the over-all efficiency of the electrical equipment at 70 per cent.

5. The train, equipped with continuous apparatus as indicated in question 4, would weigh 117 tons, and only 4 axles (instead of 8) would carry gears. Taking into account the reduced input, there would no longer be occasion to instal as much as 920-hp aggregate rated capacity of motors and equipment. To what rated capacity should this be reduced in order that the average load on each motor, taken over the entire run, should constitute the same percentage of the rated load, as in the case of question 2? Make a new estimate of the total train weight to accord with this new estimate for the aggregate capacity of the electrical equipment, which may be again taken at 19 kg per rated hp.

CHAPTER X

THE PREDETERMINATION OF THE POWER CURVE FOR A GIVEN JOURNEY

THE principles enunciated in the preceding chapters afford guidance in selecting suitable electrical equipment as regards rated hp and other general features. But as the work on a proposition advances there arrives a stage at which it becomes important to investigate, with considerable care, the application of some particular motor and equipment to the particular case in hand.

Let us take a case where it is required to operate 150-ton trains, consisting of two motor-coaches and four intermediate trailers, to a schedule speed of 25 ml ph, with 1 stop per mile. From Fig. 45 we find that the energy consumption will be 97 w hr per ton-mile. This works out at—

$$97 \times 25 = 2420 \text{ watts per ton.}$$

The average input to the train will be—

$$2{\cdot}42 \times 150 = 363 \text{ kw.}$$

From Table XXV. we obtain 70 per cent. as a rough value for the over-all efficiency of the electrical equipment. The average output from the motors is thus—

$$\frac{363 \times 0{\cdot}70}{0{\cdot}746} = 340 \text{ hp.}$$

Let the service for which we shall employ these trains be absolutely continuous for 18 hours per day—or, at any rate, let it be required that the train and equipment shall be adequately designed for such a performance. It will then be necessary that the electrical equipment shall have a *rated* capacity (on the 1-hour, 75° C. basis) of—

$$4 \times 340 = 1360 \text{ hp,}$$

since the average output for such a service is about 25 per cent. of the rated output (see p. 75).

Let this aggregate capacity be made up of 8 motors, 4 on each motor-coach. The rated capacity of each motor will thus be—

$$\frac{1360}{8} = 170 \text{ hp.}$$

For a schedule speed of 25 ml ph, with one 20-second stop per mile, we see, from Fig. 44, that the maximum speed must be 41·0 ml ph, or—

$$\frac{41{\cdot}0 \times 1609}{60} = 1100 \text{ meters per minute.}$$

Let the car wheels be of 1000 mm diameter. The speed of the car wheel is, at the maximum train speed of 41 ml ph—

$$\frac{1100}{1{\cdot}00 \times \pi} = 350 \text{ rpm.}$$

The question of the relative speed of motor armature and car axle is one of considerable importance. In general, the lower the speed of the motor, the better will be its performance, but, on the other hand, the lower its speed the greater is its weight. With the advent of interpoles it has become legitimate to employ higher motor speeds, nevertheless, the weights of equipments employing series-wound, continuous-electricity motors are not great, and it is expedient to retain fairly low motor speeds. The higher the train speed, the lower, consequently, should be the gear ratio. For locomotives required for crest speeds of 65 ml ph, or more, a 1 : 1 ratio is usually employed. On the other hand, for motor-coaches for speeds of 14 to 18 and 20 ml ph, with a stop every 0·5 to 0·8 of a mile, the gear ratio is usually taken in the neighbourhood of 3·0 to 3·5 in continuous equipments, while in single-phase equipments for such services, gear ratios as high as from 4 to 5 are taken, since otherwise the motors would be very heavy. As instances of the precise numbers of teeth on gears and pinions which are suitable in practice, Table XLVI. is of interest.

Let us for our case take a gear ratio of 2·47, providing the motor with a pinion with 21 teeth and the car axle with a gear with

$$2{\cdot}47 \times 21 = 52 \text{ teeth.}$$

Since the gear ratio is equal to 2·47, the speed of the armature when the train is running at its crest speed is—

$$350 \times 2{\cdot}47 = 865 \text{ rpm.}$$

Let us proportion the equipment to provide a speed of 20 ml ph at

the end of 15 seconds from the start, maintaining during these first 15 seconds an acceleration of—

$$\frac{20}{15} = 1{\cdot}33 \text{ ml phps.}$$

Let the last section of the rheostat be cut out at the end of the 15th second, *i.e.* when the train has acquired a speed of 20 ml ph, and let the balance of the acceleration up to 41·0 ml ph be accomplished on the "motor characteristic." If the crest speed of 41·0 ml ph is to be reached at the end of the 80th second, as shown in the diagram in Fig. 66 (which is substantially identical, although drawn to a different scale, with the diagram in Fig. 44), then the design of the motor, as regards its saturation curve, must be such as shall, with a 150-ton train equipped with eight such motors, provide the acceleration indicated in Fig. 66 for the time from the 15th to the 80th second.

TABLE XLVI.—DATA OF GEAR RATIOS OF SEVERAL TYPICAL MOTORS.

Continuous or single-phase.	Railway.	Type of motor.	Rated hp of motors.	Speed of motor at its rated (1-hr, 75° C.) load.	Diameter of driving wheels (mm).	Gear ratio.	Number of teeth.		Speed of train at rated load of motors.
				(rpm).			Pinion.	Gear.	(ml ph)
Continuous.	Lancashire and Yorkshire	Dick-Kerr	150	470	1067	1·95	22	43	30
	Central London Railway	G.E. 66A	125	660	889	3·94	15 18	59 71	17·5
	Great Northern Piccadilly & Br.	G.E. 69B	200-240	530	914	3·20	—	—	17·8
	Lancashire and Yorkshire	D.K. 5A	125	625	914	2·16	19	41	31
	Liverpool Overhead	D.K. 100A	100	570	838	2·79	19	53	20
	—	D.K. 12HT	150	640	1067	3·33	15	50	24
Single-phase.	Midland	Westinghouse	150	625	1105	3·68	19	70	22
	Midland	Siemen's	180	770	1105	2·93	30	88	34
	L.B. & S.C. Railway	A.E.G. W.E. 51	115	600	1105	4·26	—	—	18·2
	Hamburg-Blankenese	A.E.G. W.E. 51	115	600	1000	4·22	—	—	16·7
	—	G.E.A. 603 Compensated series	100	650	1067	3·74	19	71	22

We have seen in Chapter VIII. that the tractive force required to impart an acceleration of 1·0 ml phps is 49·5 kg per ton. Consequently,

our 150-ton train will, during the first 15 seconds, require a tractive force of—

$$1{\cdot}33 \times 49{\cdot}5 = 65{\cdot}9 \text{ kg per ton}$$

for acceleration, and a further 6·0 kg per ton to overcome train-friction. Thus, the total tractive force will be—

$$65{\cdot}9 + 6{\cdot}0 = 71{\cdot}9 \text{ kg per ton.}$$

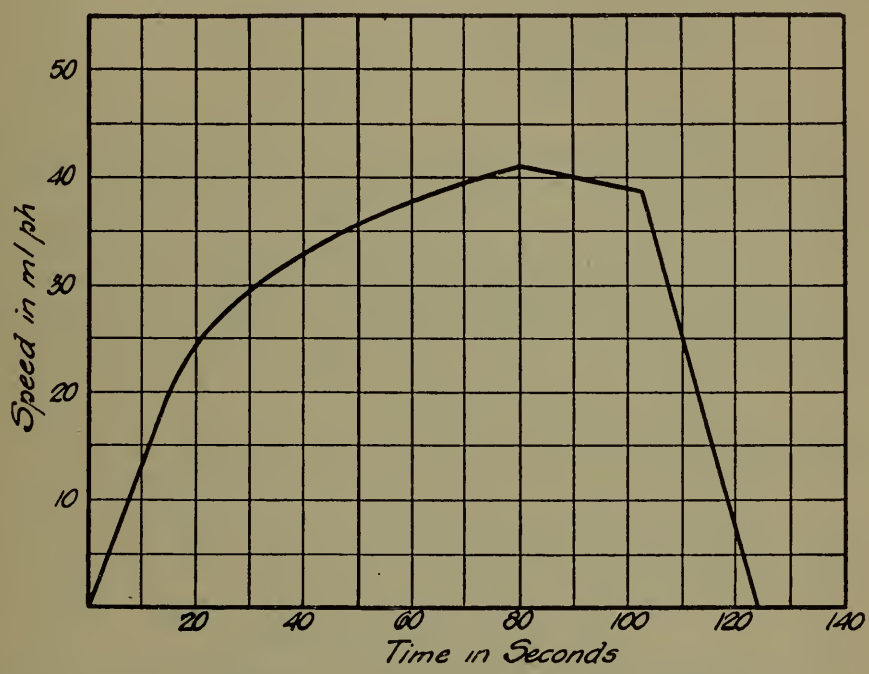

FIG. 66.—Speed-time Diagram for Run of one mile at a Schedule Speed of 25 ml ph.

The tractive force to be provided by each of the 8 motors is—

$$\frac{71{\cdot}9 \times 150}{8} = 1350 \text{ kg.}$$

At 20 ml ph, *i.e.* $\left(\frac{20 \times 1609}{3600} = \right)8{\cdot}9$ m ps, the output per motor will be—

$$1350 \times 8{\cdot}9 \times 9{\cdot}81 = 118{,}000 \text{ watts}$$

or

$$\frac{118{,}000}{746} = 158 \text{ hp.}$$

At this point, when the speed is 20 ml ph, the last resistance section has been cut out and the motor is directly across the 600-volt line. The efficiency of a series-wound, continuous-electricity motor of this rating is, including gearing, usually between 85 and 88 per cent. at

all loads between half-rated load and 50 per cent. overload. But taken over the period while electricity is on, the efficiency will be more of the order of 80 per cent. Taking the efficiency as 80 per cent. the input is—

$$\frac{118{,}000}{0{\cdot}80} = 148{,}000 \text{ watts,}$$

and the current to the motor is—

$$\frac{148{,}000}{600} = 246 \text{ amperes.}$$

This is approximately the value of the current for the first 15 seconds, although it will have fluctuated by equal amounts, above and below this value, in passing from notch to notch, *i.e.* in the process of cutting out, step by step, the resistance in series with the motor. We see from Fig. 66 that in the interval from the 15th to the 20th second it is desired that the speed shall increase from 20 to 24·5 ml ph, the mean acceleration thus being $\left(\frac{4{\cdot}5}{5{\cdot}0} = \right)0{\cdot}90$ ml phps.

The average tractive force per ton will consequently be only—

$$0{\cdot}90 \times 49{\cdot}5 + 6{\cdot}0 = 44{\cdot}5 + 6{\cdot}0 = 50{\cdot}5 \text{ kg.}$$

The average speed during this 5-second interval will be—

$$\frac{20{\cdot}0 + 24{\cdot}5}{2} = 22{\cdot}3 \text{ ml ph,}$$

$$\text{or} \qquad \frac{22{\cdot}3 \times 1609}{3600} = 9{\cdot}9 \text{ m ps.}$$

The average output per motor is equal to—

$$\frac{50{\cdot}5 \times 150}{8} \times 9{\cdot}9 \times 9{\cdot}81 = 91{,}800 \text{ watts.}$$

The average current input per motor is equal to—

$$\frac{91{,}800}{0{\cdot}80 \times 600} = 191 \text{ amperes.}$$

Making similar calculations for the average current input for each successive 5 seconds, the results set forth in column H of Table XLVII. are obtained, and these values are plotted in Fig. 67, which accordingly gives the current flowing to each motor from the first instant of starting up to the end of the 80th second, *i.e.* up to the moment when the supply of electricity is cut off.

TABLE XLVII.—ESTIMATION OF THE CURRENT INPUT TO EACH MOTOR AND TO THE TRAIN.

A	B	C	D	E	F		G	H	J	K	L
Interval (seconds).	Increase in speed (ml ph).	Mean acceleration during interval (ml ph ps).	Total tractive force (including friction) (kg per ton).	Tractive force per motor (kg per ton).	Mean speed during interval. (ml ph).	(m ps)	Output of each motor (watts).	Current input per motor (amperes).	Total current to train (amperes).	Total output from all motors (kw).	Total input to train (kw).
At end of 15 seconds	(20·0)	1·330	71·9	9·00	20·0	8·9	118,000	246	1,968	944	1,180
15–20	4·50	0·900	50·5	6·31	22·3	9·9	91,800	191	1,528	734	918
20–25	3·00	0·600	35·7	4·46	26·0	11·6	76,100	158	1,264	609	761
25–30	2·20	0·440	27·8	3·35	28·6	12·8	63,000	132	1,056	504	630
30–35	1·90	0·380	24·8	3·10	30·6	13·7	62,500	130	1,040	499	625
35–40	1·50	0·300	20·9	2·61	32·3	14·4	55,300	115	920	442	553
40–45	1·30	0·260	18·9	2·36	33·7	15·0	52,100	109	872	416	521
45–50	1·20	0·240	17·9	2·24	35·0	15·6	51,500	107	856	411	515
50–55	1·05	0·210	16·4	2·05	36·1	16·1	48,500	101	808	388	485
55–60	0·95	0·190	15·4	1·93	37·1	16·6	47,200	98·5	788	378	472
60–65	0·90	0·180	14·9	1·86	38·1	17·1	46,900	97·7	782	375	469
65–70	0·87	0·175	14·7	1·83	38·9	17·4	46,800	97·5	780	374	468
70–75	0·83	0·166	14·2	1·78	39·8	17·8	46,600	97·1	777	373	466
75–80	0·80	0·160	13·9	1·74	40·6	18·1	46,400	96·6	773	371	464
	41·00										

Out of the first 15 seconds, however, the motors were, for the first 7·5 seconds, arranged in four groups, each consisting of two motors in series, as shown in Fig. 68.

During the succeeding 7·5 seconds they were arranged in eight parallel branches, as shown in Fig. 69.

This is termed "series-parallel control," and serves to improve the efficiency during starting, since the rheostatic loss during the

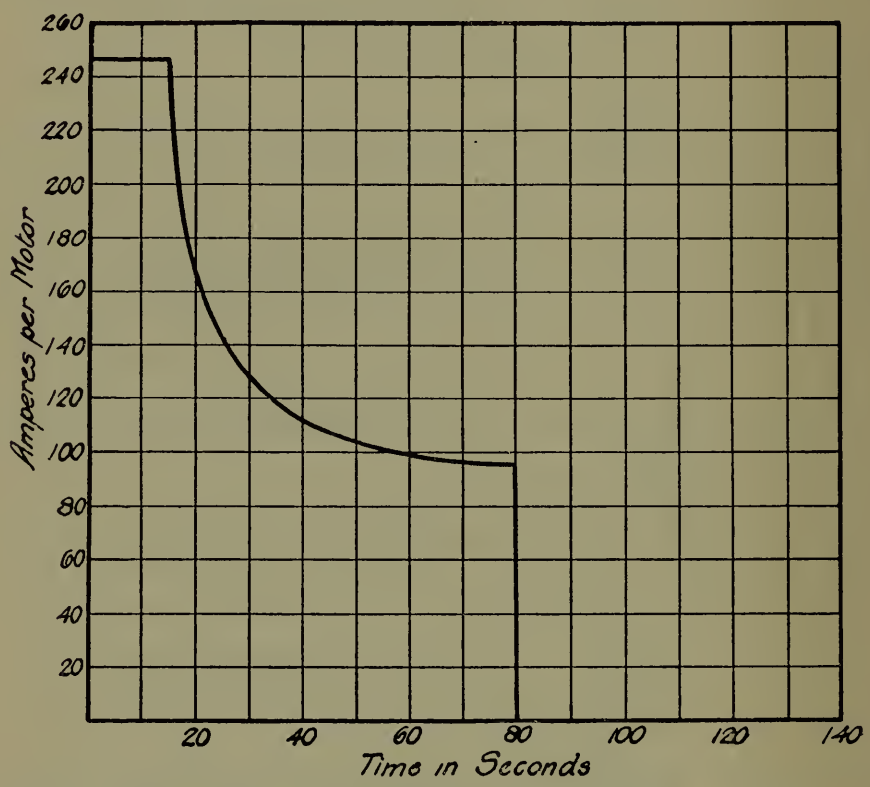

FIG. 67.—Current Input to each Motor for the Run indicated in Fig. 66.

first 7·5 seconds would be much greater were the entire eight motors all in parallel from the start.

The average current to each motor being approximately constant during the first 15 seconds, it follows that the total current supplied to the train while the motors are in series must be half that supplied when the motors are in parallel. We can therefore set down the average current input to the train at (246 × 8 =)1968 amperes while the motors are in parallel, and 984 amperes while they are in series.

When the motors are running on the motor curve, connected right across the 600-volt line without rheostats in circuit, the current input to the train is, of course, eight times that to each motor. The

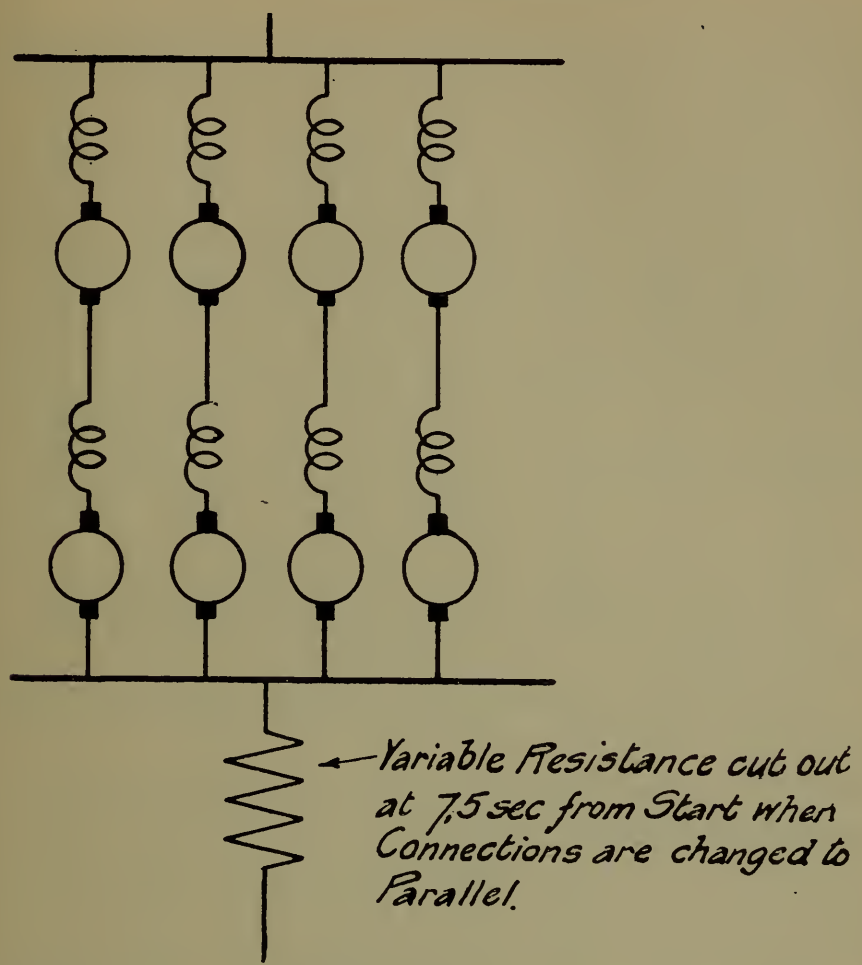

FIG. 68.—Representation of Series Arrangement of the 8 Motors on the Train.

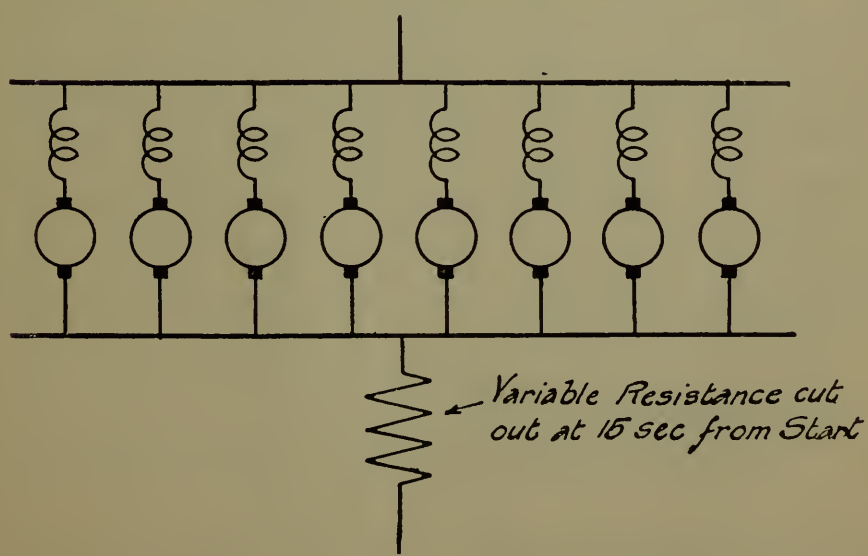

FIG. 69.—Representation of Parallel Arrangement of the 8 Motors on the Train.

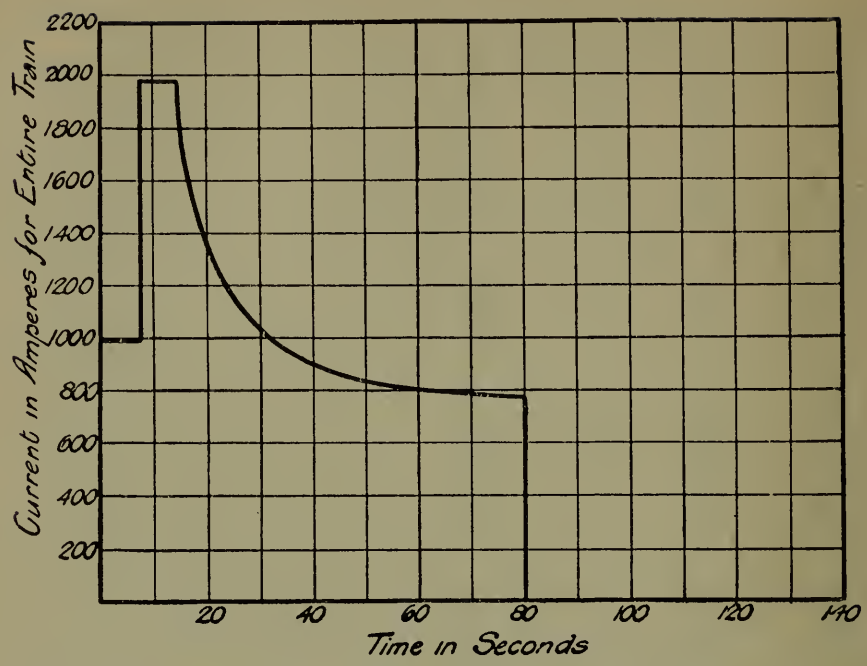

FIG. 70.—Current Input to the Entire Train for the Run Indicated in Fig. 66.

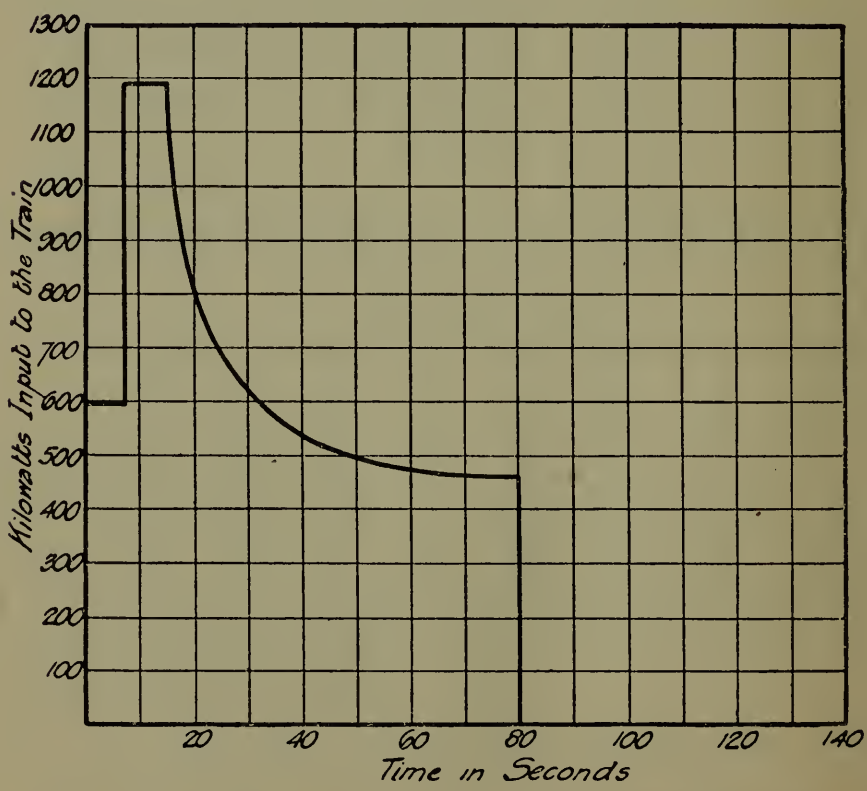

FIG. 71.—Kilowatts Input to Train for the Run Indicated in Fig. 66.

values are given in Fig. 70, plotted from column J, Table XLVII. The input to the train at any instant can be obtained by multiplying the corresponding values of the current by 600, the pressure in volts, or by estimating from the output of each motor at the assumed efficiency. Since the current input to the train during the "series" period was one half that for the "parallel" period, the power input to the train will vary in a similar manner, and we obtain the curve of total input to the train as plotted in Fig. 71.

By integrating this curve we obtain 50,000 kw seconds, which reduces to 93 w hr per ton-mile. This is 4 per cent. less than the 97 w hr obtained from Fig. 45, for the same schedule, and the discrepancy is due to taking the value of 80 per cent. as the efficiency throughout, whereas had this figure been taken at $\left(\frac{93}{97} \times 80 = \right)76{\cdot}7$—or say 77—per cent., the two values for the energy consumption would have agreed; 77 per cent. is consequently the average efficiency *of the motor* taken over the entire 80 seconds from the start up to cut off.

Thus the total loss in the motors must be $(100 - 77 =)23$ per cent. of the total input. This leaves $(30 - 23 =)7$ per cent. for rheostatic loss, since the over-all efficiency of the electrical equipment has been assumed as 70 per cent. These different losses are shown in Fig. 72 as percentages of the total input.

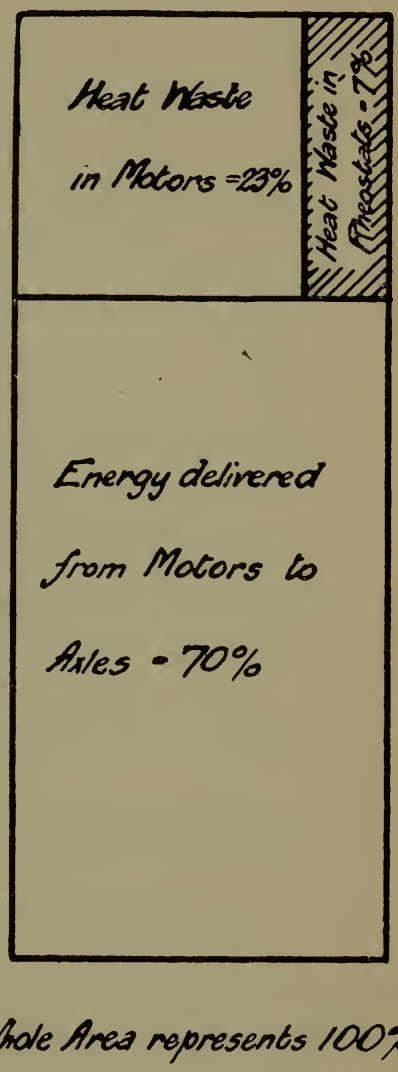

Fig. 72.—Showing Allocation of the Energy Input of the Run Indicated in Fig. 66, with an Over-all Efficiency of Electrical Equipment of 70 per cent.

Returning to Fig. 67, the curved portion gives us data for calculating the "speed-curve" of the motor which we require. This portion is reproduced in Fig. 73, in which is also drawn a curve of the train speed in ml ph taken from Fig. 66.

From the curves in the above figure we may construct the curve in Fig. 74, with speed in ml ph as ordinates and current in amperes

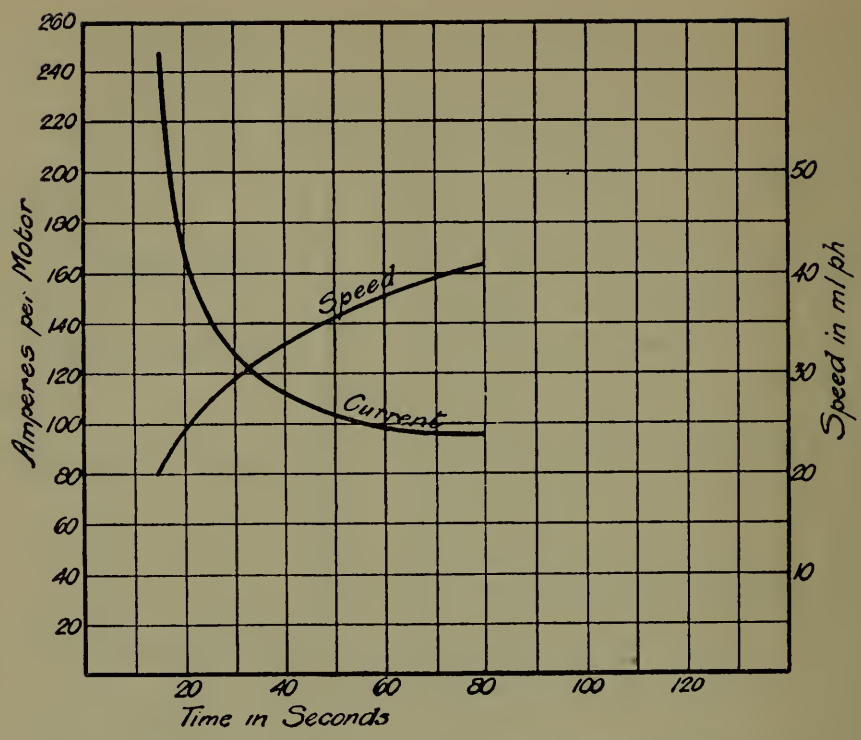

Fig. 73.—Current and Speed-Curves, re-plotted from Figs. 66 and 67.

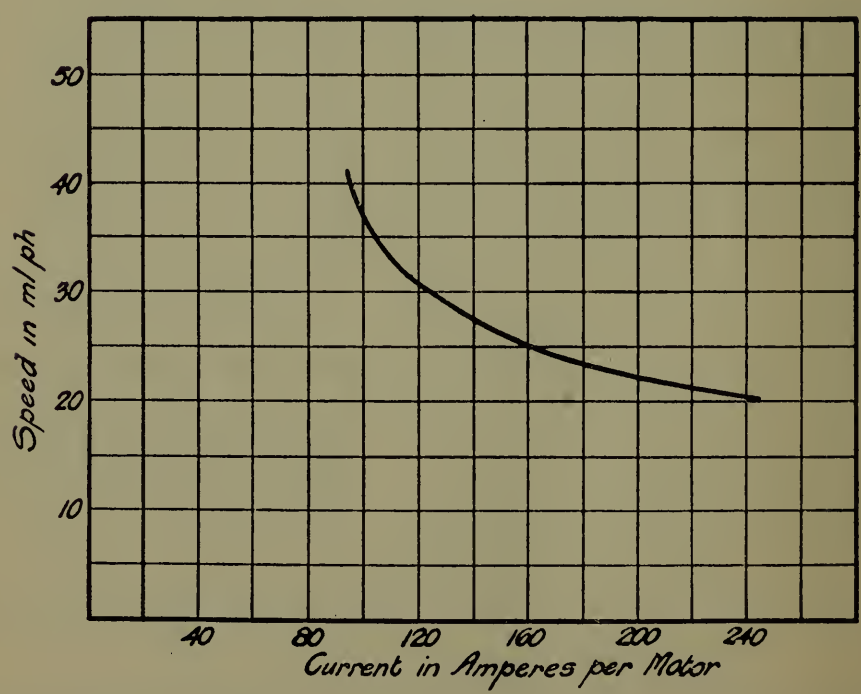

Fig. 74.—Characteristic (ml ph) Speed-Curve of the Train obtained from Fig. 73.

as abscissae. This is done by taking pairs of readings on both curves for various times. With the data that the car wheel diameter is 1000 mm, and that the ratio of gearing is 2·47, we can now plot another curve as in Fig. 75, with the speed of the motor in rpm as ordinates and with the current in amperes as abscissae.

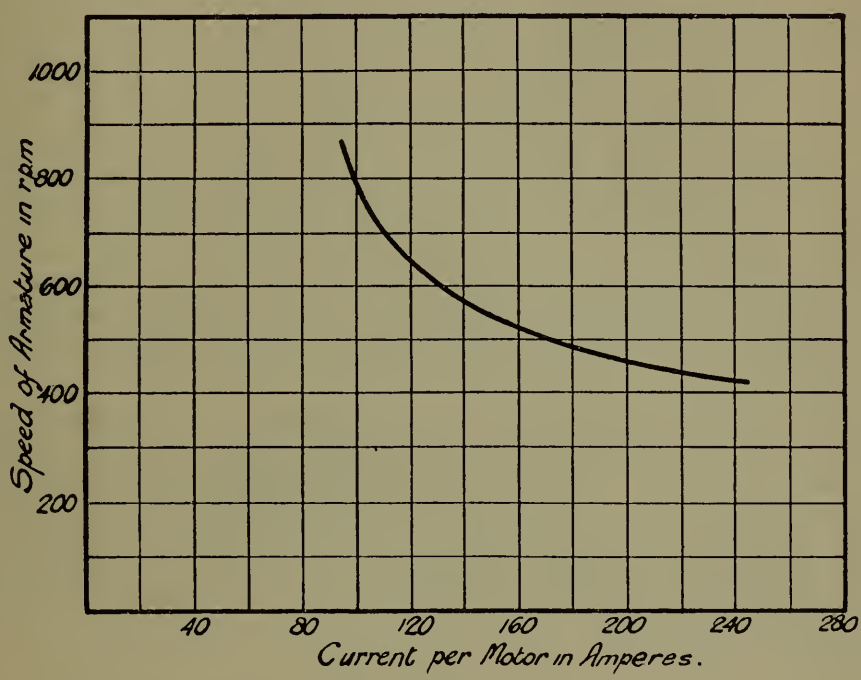

Fig. 75.—Characteristic (rpm) Speed-Curve of the Motors.

The values for these conversions are set forth in Table XLVIII., together with the calculations for obtaining the successive curves.

Table XLVIII.—Derivation of the Motor Speed-Curve.

Time (seconds).	Current input to motor (amperes).	Corresponding speed (ml ph).	Speed (meters p min).	Circumference of driving wheel (meters).	Speed of wheel (rpm).	Speed of armature (rpm).
15	246	20·0	536	3·14	171	422
18	182	23·2	622		198	489
20	166	24·3	652		208	514
25	142	27·0	725		230	568
30	128	29·2	784		250	618
40	112	32·7	877		280	692
50	104	35·7	957		304	750
60	98·0	37·8	1014		323	798
70	96·5	39·5	1060		338	835
80	96·0	41·0	1100		350	865

Having obtained the speed-curve required, we can specify a motor of the rated hp required, *i.e.* 170 (see p. 133), and with a speed-curve approximating to that of Fig. 75. Such a motor is suited to the conditions of the run indicated in Fig. 66.

To determine the form of the "motor characteristic" in a speed-time diagram when the speed-curve (as Fig. 75) for the motors employed is given, the process above described is reversed. For a particular speed the current in amperes is obtained from the curve, hence at 600 volts the input in watts is known, and the watts output is then derived on multiplying by the known efficiency of the motor for the load in question. The output in kg m ps is next obtained by multiplying by 9·81 (since 9·81 w equal 1 kg m ps). Then, dividing the figure for kg m ps by the corresponding speed in m ps, the total available effort in kg is determined, and from this the kg per ton of train. Then, subtracting 6 kg per ton for the friction component, the accelerating force and acceleration are found, corresponding to the speed taken.

By assuming a time interval of 5 seconds, the change in speed produced by this acceleration is deduced, giving two points on the speed-time diagram. With the second speed obtained, the amperes are again looked up from the curve and the operations repeated.

The calculation is best done tabularly, similarly to Table XLVII., and the complete speed-time "motor curve" can then be drawn.

CHAPTER XI

THE HEYSHAM, MORECAMBE AND LANCASTER ELECTRIFIED SECTION OF THE MIDLAND RAILWAY.

THE pressure limitations of electrical equipment with continuous motors have led to endeavours to apply alternating motors to train propulsion. Both polyphase and single-phase motors have been

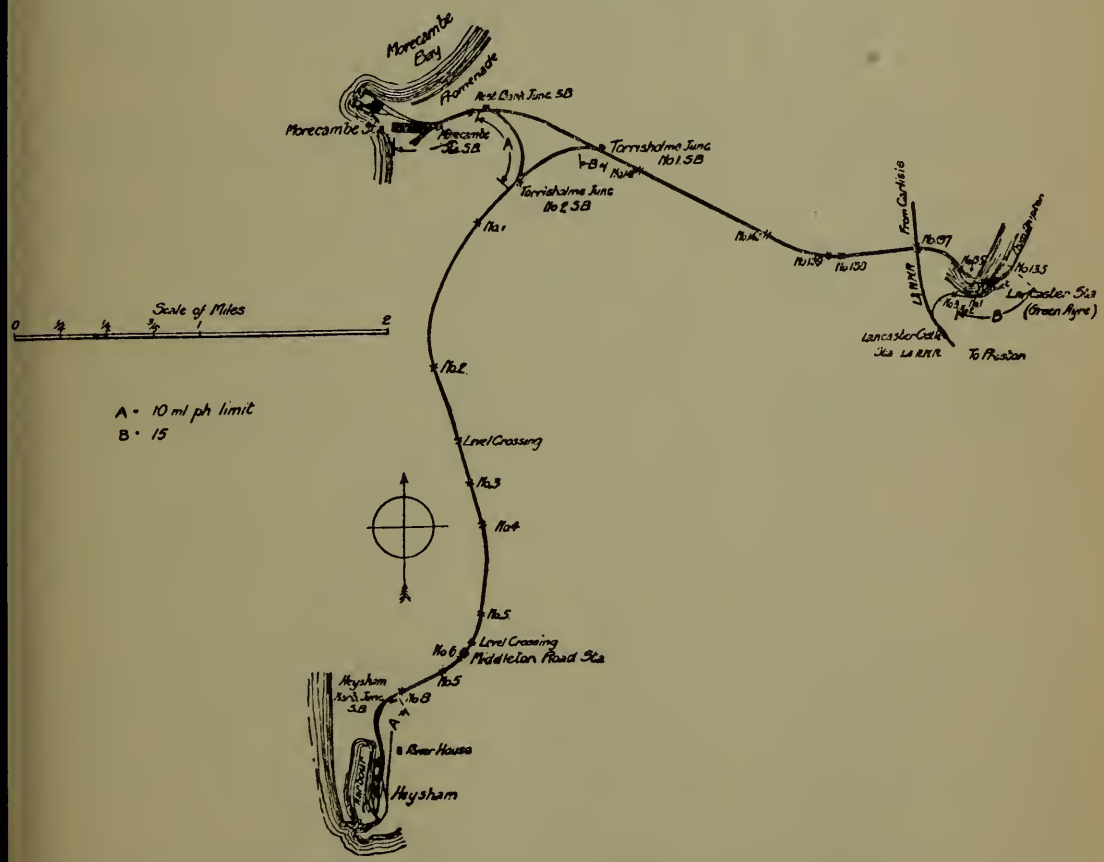

FIG. 76.—Plan of Electrified Section of Midland Railway at Heysham.

successfully employed on railways. In England motors of the single-phase type are the only alternating motors which have as

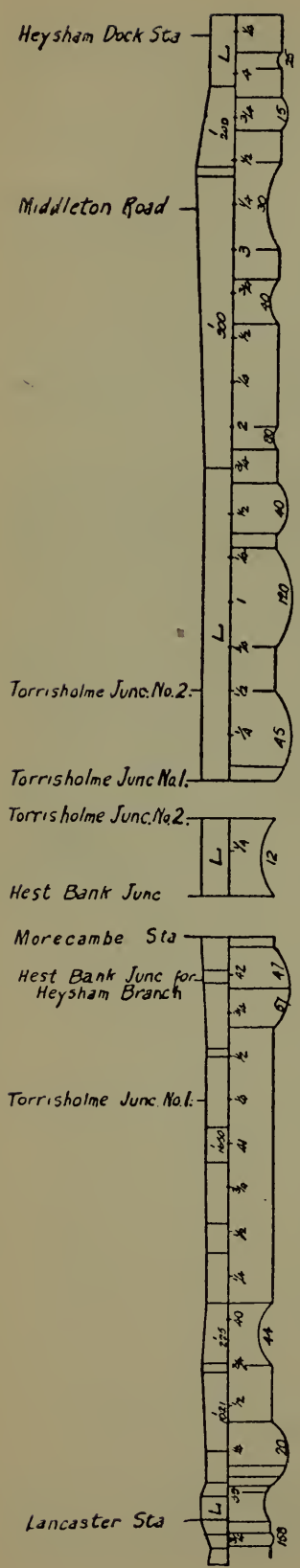

FIG. 77.—Diagram of Gradients and Curves of Electrified Portion of Midland Railway at Heysham.

yet been used. Since, so far as regards energy consumption at the train per ton-mile, single-phase equipments are about on a par with continuous equipments there has been no occasion to discuss systems in the preceding chapters, since these have dealt mainly with energy consumption. Furthermore, the wealth of available data relating to undertakings employing continuous equipments has made it convenient to base descriptions on the continuous system. There are, however, very important distinctions between the characteristics of trains, according as they are equipped with continuous or single-phase apparatus, and it is desirable at this point in the treatise to introduce data relating to single-phase trains.

The first single-phase trains operated in England are those employed on the Heysham, Morecambe and Lancaster section of the Midland Railway, where a trolley pressure of 6600 volts is used at a periodicity of 25 cycles. A plan of the electrified system is given in Fig. 76. In Fig. 77 the route is developed along a straight line, below which are indicated the gradients and curves. To obtain a broad grasp of the plan, we may consider a train operating between Heysham and Lancaster, and making an intermediate stop at Morecambe. Thus we have a route of 8·1 miles with one intermediate stop. Since Morecambe, the intermediate station, is 4·7 miles from Heysham and 3·4 miles from Lancaster, the *average* distance between stops is 4·05 miles. For our purposes we may round this off to 4·00 miles. A good many curves relating to the performance of trains operated over this route have been published, but unfortunately there are serious

evidences of inaccuracy in them. Thus, in a paper by Dalziel and Sayers, read on November 9, 1909, before the Institution of Civil Engineers (vol. clxxix. pt. I., p. 47), there are published four speed-time diagrams relating to runs between Morecambe and Heysham. I have integrated these four speed-time diagrams, and the distance works out in the four cases at 4·77 miles, 3·84 miles, 4·59 miles and 4·16 miles. The mean of these four results is 4·3 miles, the individual results being respectively 11 per cent., 12 per cent., 7 per cent., and 5 per cent. different from the mean. The largest result is no less than 26 per cent. greater than the smallest result. Similarly, two of the curves relating to runs between Morecambe and Lancaster lead to utterly different results as regards the distance between these two stations. Notwithstanding these disconcerting evidences of inaccuracy, it may, nevertheless, be of interest to examine the rough results obtained as regards the energy consumption of a train weighing 80·5 tons and operated over this route. The data is given in Table XLIX.—

Table XLIX.—Energy Consumption of Trains on the Heysham Section of the Midland Railway.

Section.	Distance (miles).	Time (seconds).	Train consumption (w hr).	Average speed (ml ph).
Heysham to Morecambe	4·7	500	13,050	33·8
Morecambe to Lancaster	3·4	368	11,800	33·2
Lancaster to Morecambe	3·4	417	11,900	29·4
Morecambe to Heysham	4·7	451	16,050	37·5
Total . .	16·2	1736 = 28·9 min	52,800	33·6

Of course, in practice the train conforms to a time-table adapted to the conditions of the traffic, which involves the times of arrival and departure of steam boats and steam trains. It is only for purposes of a rough but instructive examination of the undertaking that these four test runs have been so brought together, as in Table XLIX., as to constitute a complete round trip. If this round trip were maintained, and if at each station a 30-second stop were made, then the time, including stops, would be—

$$1736 + 4 \times 30 = 1856 \text{ seconds}$$
$$(\text{or } 28{\cdot}9 + 4 \times 0{\cdot}5 = 30{\cdot}9 \text{ minutes}).$$

The schedule speed would then be—

$$\frac{1736}{1856} \times 33{\cdot}6 = 31{\cdot}4 \text{ ml ph.}$$

Per round trip of 16·2 miles there are accomplished 16·2 train-miles, and—

$$16{\cdot}2 \times 80{\cdot}5 = 1300 \text{ ton-miles.}$$

Consequently, the energy consumption at the train works out at—

$$\frac{52{,}800}{1300} = 40{\cdot}5 \text{ w hr per ton-mile.}$$

Turning to Fig. 45, on p. 80, we find that for a schedule speed of 31·4 ml ph, with a stop every 4·00 miles, the energy consumption is given as about 38 w hr per ton-mile. The agreement is thus within 7 per cent., and it may be said that consistent values are gradually emerging from the sporadic tests being made in various parts of the world, and corresponding to various services. It is hardly justifiable to attempt to explain so small a divergence as 7 per cent., as it constitutes agreement rather than disagreement. It can, however, be partly accounted for by the frequency of sharp curves on the route, and the necessity of slowing down at some of them.

The electrical equipment comprised two 180-hp motors and the requisite auxiliary apparatus. Thus we have—

Rated capacity of equipment	. . . .	360 hp
Consumption per ton-mile	(a) . . .	40·5 w hr
Weight of train . .	(b) . . .	80·5 tons
Consumption per train-mile	($c = a \times b$) . .	3260 w hr
Schedule speed . .	(d) . . .	31·4 ml ph
Average input . .	($e = c \times d$) . .	102,000 watts

From tests given later in this chapter (see p. 154) an appropriate value for the efficiency of the equipment under these conditions of service is seen to be some 72 per cent.

∴ Average output	($f = e \times 0{\cdot}72$) . . .	73,500 watts
,, ,,	($f \div 746$)	98 hp
,, ,,	per motor	49 hp
,, ,,	in terms of rated output of motor	27·2 per cent.

As a matter of fact, the actual service is much less severe than this. Thus, it was specified that the motors should run an 80·5-ton train (one motor-coach and two trailers) for six complete double trips, allotting 20 minutes to the round trip between Morecambe and Heysham, and 15 minutes to the round trip between Morecambe and Lancaster. It would thus appear that 35 minutes would be

allotted to a complete round trip between Heysham and Lancaster, intermediate stops being made at Morecambe. But in the above analysis only 30·9 minutes were allotted to this round trip. Assuming that the additional 4·1 minutes is devoted to stops at stations, then the energy consumption remains at the figure already worked out, but the schedule speed falls to—

$$\frac{30{\cdot}9}{35} \times 31{\cdot}4 = 27{\cdot}7 \text{ ml ph}$$

as an average for the $(35 \times 6 =)210$ minutes (3·5 hours) occupied in running the six complete double trips specified. Thus, the average load was $\left(\frac{30{\cdot}9}{35} \times 27{\cdot}2 = \right)24$ per cent. of rated load, and practically conforms with the established practice on London's underground tube railways. Nevertheless, the motors are cooled by forced draught, the provision of which requires an average consumption of 750 watts per train, or 0·75 of one per cent. of the average input to the train. Moreover, the motors alone (including gear and gear case) weigh 3·12 tons each, or 17·3 kg per rated hp, whereas the C.L.R. motors (described in Chapter VI.) only weigh 1·96 tons each, or 15·9 kg per rated hp. On the G.N.P. and B. railway the G.E.69B motors employed (see Chapter VII.) have a 1-hour, 75° C. rating of 240 hp and weigh 2·80 tons each, or 11·7 kg per rated hp. No forced draught is employed with these continuous equipments. Taking the *complete* equipment in the three cases the weights are—

Railway.	Weight of complete electrical equipment per 1-hour, 75° C. rated hp.
Central London	20·0 kg
Great Northern Piccadilly and Brompton	15·4 kg
Heysham, Morecambe and Lancaster (Siemens' equipments)	40·7 kg

It will be found characteristic of single-phase railway equipments that their weight per hp is of the order of twice that of continuous equipments. The G.E.69B motor of the G.N.P. and B. railway is sold as of only 200-hp rating, being very conservatively rated, but even if taken on this basis, instead of on the 1-hour, 75° C. basis, the weight of total electrical equipment is only $\left(\frac{240}{200} \times 15{\cdot}4 = \right)18{\cdot}5$ kg per hp, as against 40·7 kg per hp for the Siemens single-phase equipments employed at Heysham. On the other hand, although the Siemens motors were sold as of 180-hp rated capacity, it is stated to have since been demonstrated that they rate at this output, having only 75° C. rise at the end of one hour, without resorting to forced ventilation, and that with forced ventilation they rate at 210 hp. Thus, comparing the forced-ventilated Siemens motors as of 210 hp,

with the G.E.69B motors, without forced ventilation, as of 200 hp, then the respective values are as follows—

Type.	Rating.	Weight of complete equipment per hp (kg).
Siemens (forced ventilation) .	210	35·0
G.E.69B (natural ventilation) .	200	18·5

On this basis the single-phase equipment weighs—

$$\left(\frac{35{\cdot}0}{18{\cdot}5} = 1{\cdot}89\right)89 \text{ per cent.}$$

more than the continuous equipment. It should also be pointed out that whereas at their rated load the G.E.69B motors have a speed of only 530 rpm, the speed of these Heysham motors at their rated load is 770 rpm, *i.e.* 45 per cent. greater, and, nevertheless, their weight per rated hp is much greater.

With this preliminary sketch of the nature and purpose of the system, the reader will be in a position to read with some interest a descriptive account of the apparatus and of the trains on which it is installed.

The rolling stock consists of three trains, each consisting of three coaches. There are three motor-coaches, two with equipments by Messrs. Siemens Brothers, and one equipped by the British Westinghouse Company. The remainder of the rolling stock consists of trailer-coaches. Each end of each of the motor-coaches and trailer-coaches is provided with control apparatus. This provision is made in order that the length of the trains may be varied to suit the traffic, which varies from season to season. The coaches are repeatedly reversed at the triangular junction at Morecambe.

A specification of both the Siemens and Westinghouse motor-coaches, the trailer-coaches, and of a typical 3-coach train, are given in Table L.

TABLE L.—SPECIFICATION OF ELECTRIC TRAINS ON MIDLAND RAILWAY (HEYSHAM, MORECAMBE AND LANCASTER BRANCH).

A.—Motor-Coach. B.—Trailer-Coach. C.—Complete Train.

A.—MOTOR-COACH.

General—	Siemens.	Westinghouse.
Total length over end panels	60 ft	60 ft
Length of passenger compartment	52 ft	—

Composed of three divisions, the centre one 25 ft in length, transverse seats, and two divisions 13 ft 5 ins with longitudinal seats.

Height, over-all, above rail level	12 ft 9 ins	12 ft 9 ins
Width, over-all, outside	9 ft	9 ft
Seating capacity	72	72

	Siemens.	Westinghouse.
Weight of coach without passengers	40·5 tons	37·5 tons
,, of coach body, including under-frame, air compressors, seats, upholstering, and all fittings	15·1	14·9
Seats per foot length of coach	1·2	1·2
,, ton weight of coach	1·78	1·92
Weight per seat	0·56 tons	0·52 tons
Trucks and Bogies—		
Weight of motor bogie without motor	6·5 tons	6·5 tons
Wheel base of motor truck	8 ft 6 ins	8 ft 6 ins
,, of trailing truck	8 ft	8 ft
Axle diameter, mid frame (motor bogie)	6½ ins	—
,, ,, at bearing	4¾ ins	—
Length of bearing	9 ins	—
Diameter of driving wheels (new)	3 ft 7½ ins	—
Gauge of track	4 ft 8½ ins	—
Electrical Equipment—		
Rated hp	180	150
Pressure in volts	340	235

Method of control: multiple unit, the Siemens coach having the "all-electric system," while the Westinghouse has the electro-pneumatic system.

Gear ratio	2·9	3·9
Weight of one motor alone	2·8	2·5
,, of motor and gear	3·12	2·78
Motors per motor-coach	2	2
Total weight of motors with gearing per motor-coach	6·25 tons	5·55 tons
Weight of balance of electrical equipment (*i.e.* controllers, transformer, rheostats, etc.) per motor-coach	8·65	6·8
Total weight of electrical equipment per motor-coach	14·9	12·4
Ratio of total weight of electrical equipment to weight of motors and gearing	2·38	2·22
Weight of motor in kg per hp (rated)	15·5	16·8
,, of motor and gear in kg per hp	17·3	18·7
,, of all electrical equipment in kg per ton weight of motor-coach	368	330
Ratio of total weight of electrical equipment to weight of motor-coach	0·37	0·33

B.—Trailer-Coach.

Length, over-all	43 ft
Height, over-all, above rail level	13 ft
Width, over-all, outside	9 ft
Seating capacity	54
Total weight of trailer-coach	17·5 tons

C.—Complete 3-Coach Train.

Number of motor-coaches	1	1
,, of trailer-coaches	2	2
Total length of train	150 ft	150 ft
Weight of motor-coach component	40·5	37·5
,, of trailer-coach component	36	36
Total weight of train without passengers	77 tons	74 tons
Seating capacity	180	180
Number of motors per train	2	2

	Siemens.	Westinghouse.
Total hp of train	360	300
,, weight of motors and gearing	6·25 tons	5·55 tons
,, ,, of electrical equipment	14·9 ,,	12·4 ,,
Rated hp per ton of train	4·68	4·05
,, per seat of train	2·00	1·67
Total weight of motors and gearing in kg per ton of train	82·5	76·2
Total weight of motors and gearing in kg per seat of train	35·2	31·2
Total weight of electrical equipment in kg per ton of train	197	170
Total weight of electrical equipment in kg per seat of train	83·6	69·5
Ratio of total weight of electrical equipment to total train weight	0·197	0·170

The braking equipment is of the combined power (vacuum) and hand type.

A photograph of a Siemens motor-coach complete is shown in Fig. 78, and of the Westinghouse motor-coach in Fig. 79. A standard 3-coach train hauled by a Siemens motor-coach is shown in Fig. 80.

The train specification issued by the engineers of the Midland Railway Company, called for two motors per motor-coach, to be carried on one bogie. Thus each motor-coach comprises a motor bogie and a bogie without motors. The normal train was specified to consist of one motor-coach and two trailers, the motor-coach seating 72 passengers and the two trailers to each seat 54 passengers, thus providing 180 seats per train of three coaches.

It will be seen from Fig. 77 that the route embodies several sharp curves. The speed of the train is restricted to 15 ml ph at these curves, and in some instances to 10 ml ph. The contractors were called upon to supply trains capable, under these conditions, of maintaining a 20-minute service between Heysham and Morecambe with a single train, and a 15-minute service between Morecambe and Lancaster. It was required that the motor-coach should be of such capacity as to enable it on occasions to haul two additional standard Midland Railway coaches, each weighing 26 tons, bringing up the total train weight to some 125 to 130 tons. This train was to be capable of climbing the gradient of 1 in 70 existing on the short, single-track branch line between Lancaster Green Ayre Station and Lancaster Castle Station.

To meet these conditions, Messrs. Siemens Bros. equipped their two motor-coaches with two 180-hp motors per coach, and the Westinghouse Co. equipped their motor-coach with two 150-hp motors. The specification required that the motors were to be capable of delivering, when tested on the stand with single-phase electricity and at the specified periodicity of 25 cycles per second, their declared output for one hour with a temperature

Fig. 78.—Siemens Motor-Coach Complete.
(*Heysham Branch of the Midland Railway.*)

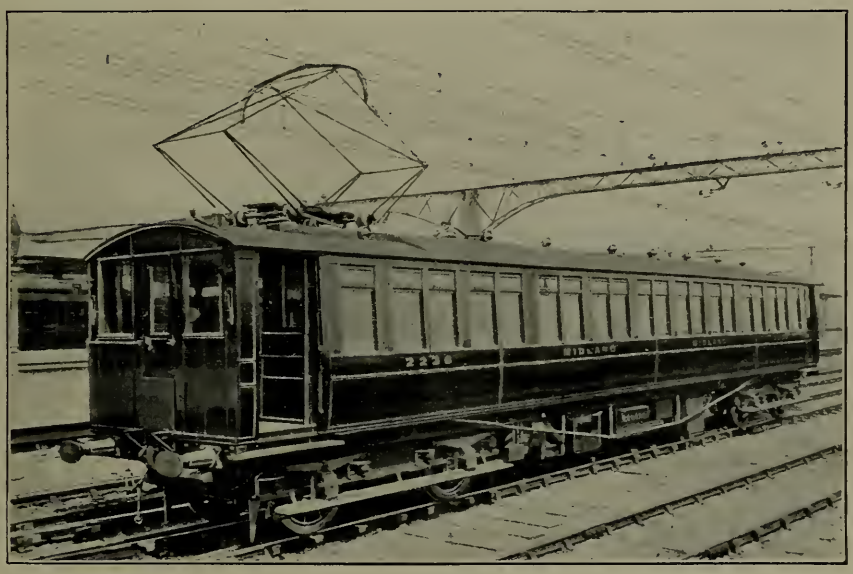

Fig. 79.—Westinghouse Motor-Coach Complete.
(*Heysham Branch of the Midland Railway.*)

Fig. 80.—View of Train consisting of Siemens Motor-Coach and Two Trailers.
(*Heysham Branch of the Midland Railway.*)

[*To face p.* 152.

rise not exceeding 135° F. (57·2° C.) above the surrounding air. They were also required to have a temperature rise not exceeding 90° F. (32·2° C.) after hauling the three-coach train for six double trips at the schedule mentioned above, from Heysham to Morecambe, Morecambe to Lancaster and return. Six such trips, however, only correspond to carrying 24 per cent. of the rated load of the motors for three and one-half hours.

The 20-minute service between Heysham and Morecambe requires that the train shall perform the single journey in about 500 seconds from start to stop, the train making on an average 6 journeys either

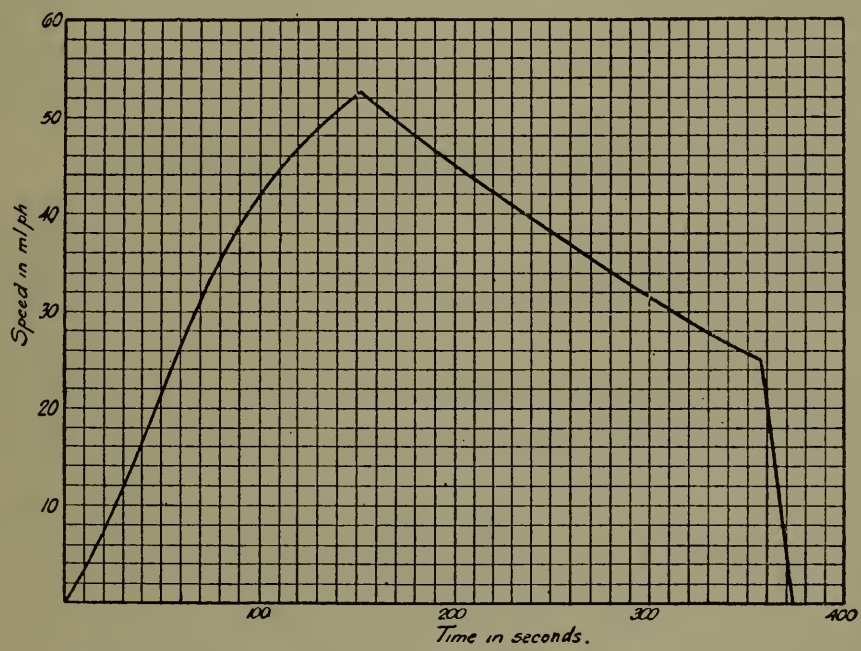

Fig. 81.—Speed-Time Diagram for 3·43-miles Run at an Average Speed of 33·3 ml ph.

way in each hour. The distance is about 4·7 miles, and thus the train performs its trip at an average speed of some

$$\frac{3600}{500} \times 4{\cdot}7 = 33{\cdot}8 \text{ ml ph.}$$

Between Morecambe and Lancaster the time taken by a train per trip is about 400 seconds, there being 8 such trips in each hour. The average speed thus amounts to about—

$$\frac{3600}{400} \times 3{\cdot}4 = 30{\cdot}6 \text{ ml ph.}$$

For so long a run, the speed-time diagram is less definite than that for the short runs which we have considered in previous chapters. Nevertheless, the speed-time diagram of Fig. 81 has been

drawn as a mean of Figs. 25 and 27 of Dalziel and Sayers' original paper, in order to form a basis for analysis, these figures representing runs between Morecambe and Lancaster and between Lancaster and Morecambe respectively.

The estimation shown in Table LI. is based on the speed-time diagram of Fig. 81.

TABLE LI.—ANALYSIS OF TESTS ON THE MIDLAND TRAINS.

Distance from start to stop	3·43 miles
Time from start to stop	373 seconds
Average speed start to stop	33·3 ml ph
Assumed duration of stop for a hypothetical schedule	60 seconds
Schedule speed	28·7 ml ph
Crest speed	52·5 ml ph
Momentum per ton at crest speed = $0{\cdot}0278 \times 1{\cdot}09 \times V^2$	83·5 w hr
,, ,, ton-mile	24·2 w hr
Assumed tractive force per ton for train friction. (This is taken lower than for runs with frequent stops, as in accordance with experience, and more especially because in this case only two out of 12 axles carry gears)	4 kg
Time electricity is on	152 seconds
Mean speed during this time	32 ml ph
Distance covered during this time	1·35 miles
Train friction per ton while electricity is on	8700 kg m
,, in w hr	123·6 w hr
,, per ton-mile	6·9 w hr
Input per ton-mile allocated to momentum and friction	31·1 w hr
Total input per ton-mile	43·2 w hr
Balance to be accounted for as loss in the electricity equipment	12·1 w hr
Efficiency of electrical equipment as thus estimated	72 per cent.

The train weighed 80·5 tons, and comprised 3 coaches, of which one was a Siemens motor-coach weighing 40·5 tons. The remaining (80·5 − 40·5 =)40 tons was made up of two trailers and the load.

The 40·5 tons weight of the Siemens motor-coach was made up as follows—

Motors with gear, gear-case, and suspension-bar	6·25
Main transformer	2·725
Auxiliary commutating transformer and preventive coil	0·975
Pumps and compressors	0·475
Contactors and chambers	1·125
Other sundries, including bows, blowers, controllers, cables, etc.	3·35
	14·9
Coach-body	13·25
Special supports	1·3
Coach-bogie	4·5
Motor-bogie	6·55
	40·5

Let us now continue the tabular calculation of Table LI—

Input per ton-mile	43·2 w hr
„ „ train-mile	3480 w hr
Average input per train $= \frac{3480 \times 28{\cdot}7}{1000} =$ . .	100 kw.
„ „ „ motor	50 kw
„ output per motor = 36 kw = . .	48·2 hp
„ in per cent. of rated output of 180 hp .	26·8 per cent.
„ output per motor during the time electricity is on $= \frac{373 + 60}{152} \times 48{\cdot}2 =$ . . .	137 hp

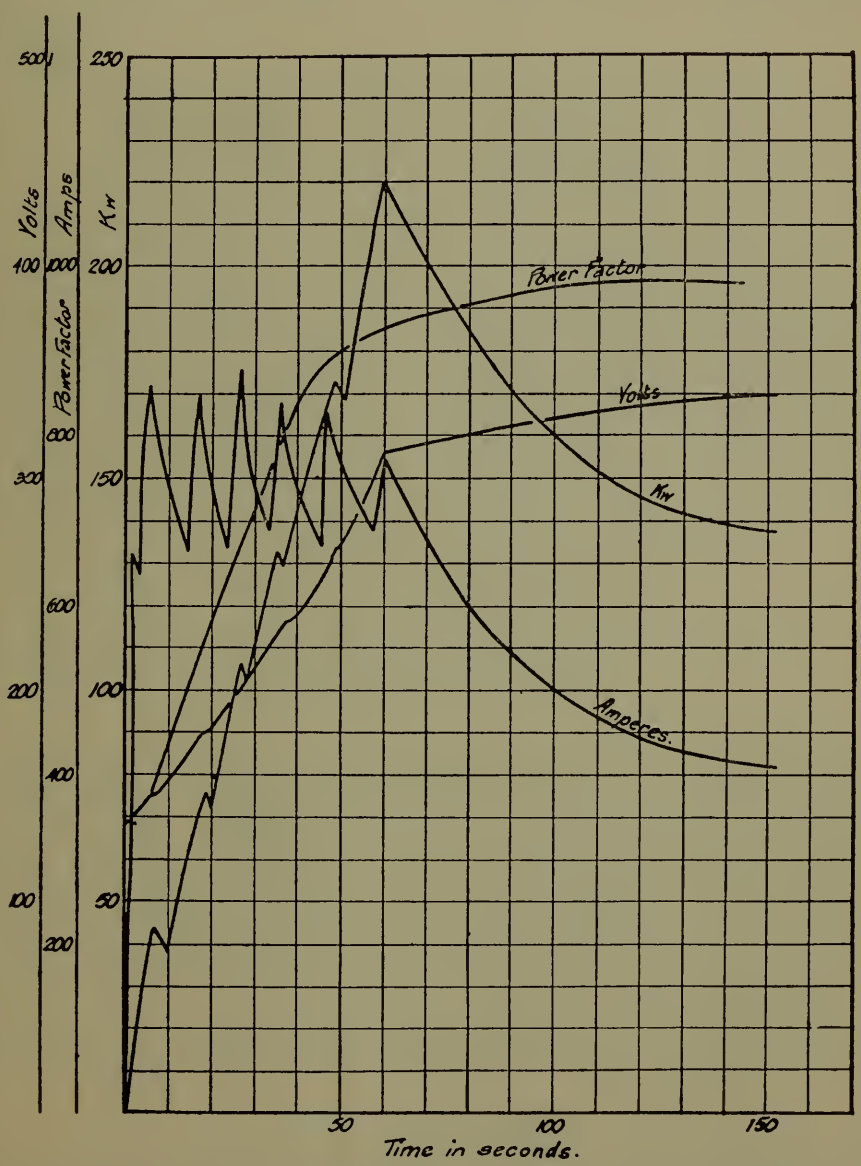

FIG. 82.—Curves relating to the Energy Input for the Run indicated in Fig. 81.

In Fig. 82 are given the curves of pressure and current per motor during the 152 seconds that electricity is on, as also of the values of the kw input and the power-factor during the run.

The characteristic curves of the motor, as obtained on factory tests, are shown in Fig. 83.

In both the Siemens and Westinghouse motor-coaches the motors are provided with forced draught. For the Siemens motor-coach the suction duct has been carried inside the coach under one of the seats, the whole of the air coming in this case from the inside of the coach. The Westinghouse motor-coach has a similar duct inside, but there is also provided a suction duct with a filter taking air from outside

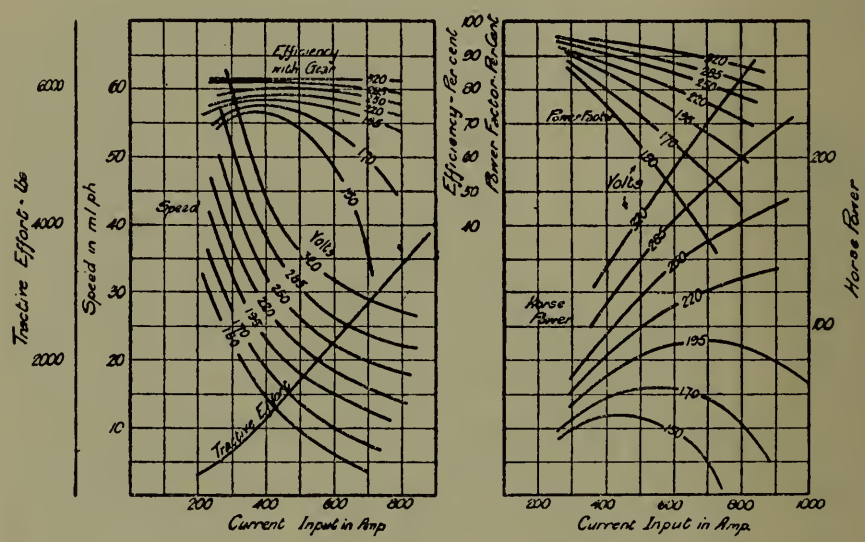

FIG. 83.—Characteristic Curves of the Siemens Motor on the Heysham Electrified Portion of the Midland Railway.

the coach. The power required by the fan motor is 1·5 kw in the Westinghouse, and 0·75 kw in the Siemens coach. The tests of the Siemens motor at the makers' works, with forced draught and with 300 volts at the terminals (the full pressure being 340 volts), is stated to have indicated that they were capable of sustaining 210 hp for one hour without exceeding their guaranteed temperature-rise. It is also stated that the motors were capable of developing 150 hp at full pressure and with a forced draught that requires about 1·5 kw for the fan, without exceeding their guaranteed temperature rise.

The weight of the Westinghouse motor-coach is 37·5 ton, this weight being made up as follows (weights are given in tons):—

Motors with gear, gear-case, and suspension bar	5·55
Main transformer	2·55
Auxiliary commutating transformer and preventive coil	0·47
Pumps and compressors	0·80
Contactors and chambers	0·49
Other sundries, including bows, blowers, controllers, cables, etc.	2·49
	12·35
Coach-body	13·25
Special supports	0·85
Coach-bogie	4·50
Motor-bogie	6·55
	25·15
Total	37·5

The Heysham electrified section of the Midland Railway has been described in detail in a great many articles in technical journals and in papers read before various institutions. To these articles and papers the reader is referred for more complete details, not only of the electrical equipment of the coaches, but also of the overhead construction, power station, etc. For this purpose I have brought together the following references :—

1. *Electrical Engineering*, for June 11 and 18, 1909.
2. *Electrician*, for June 12 and 19, 1908.
3. *Railway Gazette*, for June 12 and 19, 1908.
4. *Engineer*, for June 12 and 19, 1908.
5. *Electrical Review*, for June 12 and 19, 1908.
6. *Light Railway and Tramway Journal*, for June 5, 1908.
7. *Journal Institution of Electrical Engineers.* Paper on "Single-Phase Railways," by Percy C. Jones, vol. 43, p. 723.
8. *Proceedings of the Institution of Civil Engineers*, vol. clxxix. pt. 1, p. 31. "The Single-Phase Electrification of the Heysham, Morecambe and Lancaster Branch of the Midland Railway," J. Dalziel and J. Sayers.

CHAPTER XII

THE HEATING OF RAILWAY MOTORS

A VERY useful comparison of railway motors may be based on a study of the watts dissipated in internal motor losses per ton weight of motor. This comparison may be made under the conditions of actual service and also under the conditions of the 1-hour, 75° C. rating. In carrying out such a comparison we must exclude the gear losses, and also the weight of the gear and gear case. It is reasonable that the results obtained should be quite varied, since they will to a considerable extent be dependent upon the design of the motors and upon the conditions attending their use in actual service. Nevertheless, a general knowledge of the order of magnitude of the "watts per ton" is of importance.

G.E.66A. Motor—Central London Railway

Let us take, for example, the motor used on the Central London Railway which was considered in Chapter VI. It is rated at 125 hp, and from its efficiency curve (curve F of Fig. 52) the efficiency at this output, including gear loss, is seen to be 90 per cent. The efficiency, exclusive of gearing, is seen to be 93·5 per cent. (curve E of Fig. 52). Therefore the gearing loss is some 3·5 per cent. The input to the motor at rated load is—

$$\frac{125}{90} \times 746 = 103{,}800 \text{ watts.}$$

Output from the armature axle to the gear—

$$= 0{\cdot}935 \times 103{,}800$$
$$= 97{,}000 \text{ watts.}$$

Internal losses $= 103{,}800 - 97{,}000$
$= 6800$ watts.

The weight of the motor without gear and gear case is 1·75 ton. The loss per ton, therefore, amounts to—

$$\frac{6800}{1{\cdot}75} = 3880 \text{ watts.}$$

Similarly, the internal losses in the motors may be estimated in other cases, and three examples are given in Table LII.—

TABLE LII.—VALUES OF INTERNAL LOSSES IN MOTORS AT RATED LOAD.

Motor.	Railway.	Weight of motor (tons).	Rated output (hp).	Efficiency, including gear at rated load.	Input to motor (watts).	Efficiency, excluding gear at rated load.	Output from motor to gear (watts).	Internal loss (watts).	Internal loss (watts per ton of motor).
G.E. 66A	Central London	1·75	125	90·0	103,800	93·5	97,000	6,800	3,880
G.E. 69B	Gt. Northern Piccadilly and Brompton	2·51	200	88·0	169,500	92·0	156,000	13,500	5,390
			240 (1 hr 75°)	87·0	206,000	91·0	187,500	18,500	7,370
Dick-Kerr	Lancashire and Yorkshire	2·75	150	88·0	127,000	92·0	117,000	10,000	3,640

A representative figure for the watts loss per ton at rated output for continuous railway motors is 4500.

The G.E.69B motor of the G.N.P. and B. Railway is an instance of a case where this value is greatly exceeded. The motor is usually sold as a 200-hp motor, but on the 1-hour, 75° C. basis, it is rated at 240 hp. It has openings in the frame protected by perforated covers, as shown in Fig. 84, whereas the C.L.R. and the L. and Y. motors are completely enclosed.

In Chapter VI. a series of tests made on the Central London Railway were analysed, and we can use the results to study the average loss in the motors taken over the period of actual service. Certain of the results, as calculated in Chapter VI., Table XXIX., are re-stated below.

Total number of coaches.	7	6	5	4	3
Average load per motor while electricity is on (hp).	56·8	49·5	43·4	35·6	30·0

Referring to the efficiency curve of Fig. 52, we can read off the efficiencies, exclusive of gearing, corresponding to the above loads, from the curve E.

These values are set forth in Table LIII.—

TABLE LIII.—CALCULATIONS FOR C.L.R. MOTOR LOSSES IN SERVICE.

Average load per motor while electricity is on (hp)	56·8	49·5	43·4	35·6	30·0
Efficiencies, including gear, corresponding to above loads (from Fig. 52)	86·8	85·5	84·0	82·0	79·5
Corresponding input to motor, including gear loss (watts)	48,930	43,100	38,600	32,400	28,190
Estimated efficiency, excluding gear (from Fig. 52)	94	94	94	93·5	93·3
Average output from the motor to the gear (watts)	46,000	40,510	36,280	30,300	36,300
Average loss in motor while electricity is on (hp)	2,930	2,590	2,320	2,100	1,890
Ratio of time electricity is on to the total time of each run, including stop (see Chapter VI., p. 89)	$\frac{48}{110} = 0{\cdot}436$				
Average loss per motor (watts) taken over the whole run of 110 seconds	1,280	1,130	1,010	915	825
Average loss per motor taken over the whole running period (watts per ton)	730	646	577	522	470

The value for the 6-coach train is 646 watts per ton. This is only 16·6 per cent. of the 3880 watts per ton, at rated load, as set forth in Table LII.*

G.E.69B. Motor—Great Northern, Piccadilly, and Brompton Railway

The tests on the Great Northern Piccadilly and Brompton line which were considered in Chapter VII. may also be analysed in a similar way. In Table LII. the values of 5390 watts per ton and 7370 watts per ton are taken as the loss in the G.E.69B motor when rated at 200 hp and 240 hp respectively.

From Table XXXIII. we have obtained values for the average hp output during two test runs. These values are—

Test.	A	B
Average hp output from each motor during the time electricity is supplied	100	114

* While the calculations given in this chapter have considerable interest from the standpoint of *relative* comparisons, the absolute *quantitative* results for the watts per ton under the conditions of service are considerably too low, since they are arrived at by employing the efficiencies for the average load when the motors have full line pressure at their terminals, whereas actually the motors are for a part of the time in series and there is also resistance in circuit. Under these conditions the average efficiency of a motor is considerably lower, and its internal loss is consequently higher, often to the extent of 50 per cent. or more.

The efficiencies at these loads can be obtained from the curve of Fig. 57, and the motor losses may be estimated as shown in Table

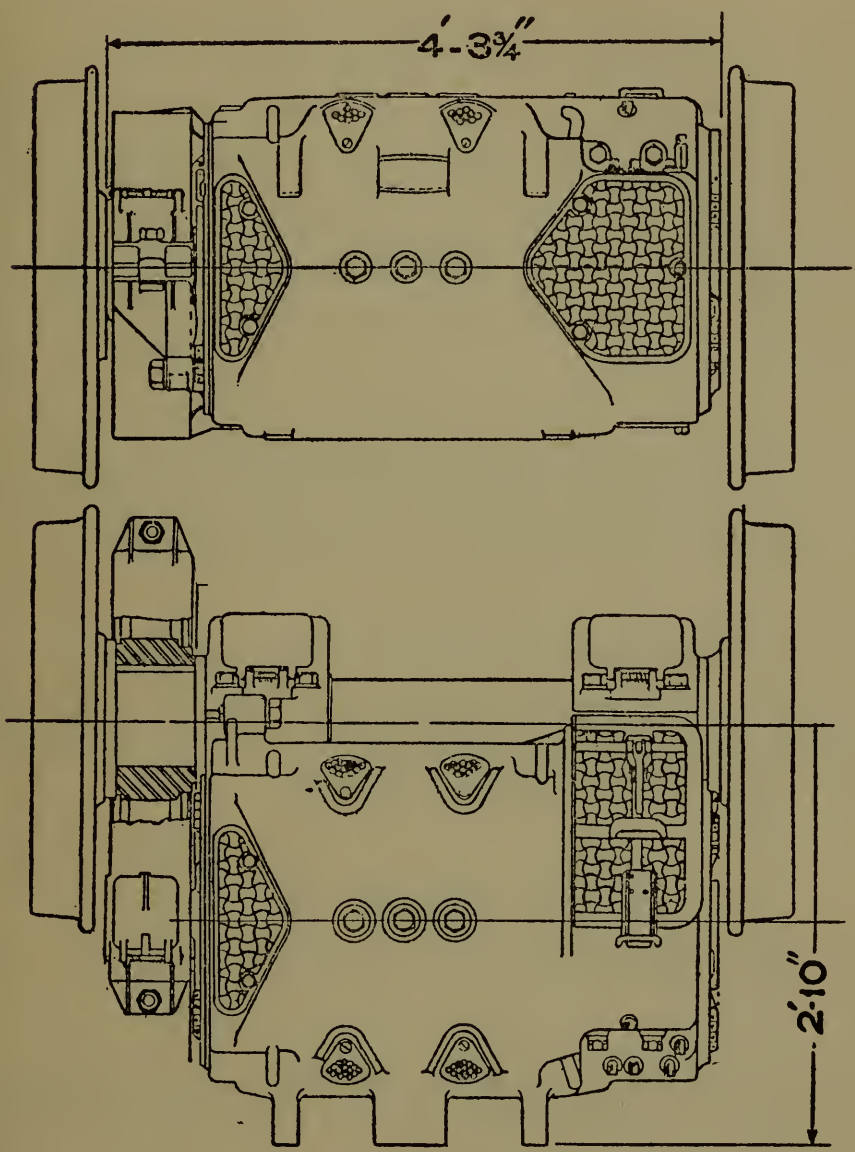

Fig. 84.—G.E.69B Series-wound Continuous Motor.

One-hour rating = 240 hp
Weight (including gearing) = 2·8 tons
Speed at one-hour rating = 530 rpm

LIV. The weight of the motor, exclusive of gear and gear case, is 2·51 tons.

TABLE LIV.—CALCULATIONS OF G.N.P. AND B. RLY. MOTOR LOSSES IN SERVICE.

Tests.	A	B
Average hp output per motor while in circuit	100	114
Efficiency of motor (including gear) at the above load, from Fig. 57	89	89·5
Corresponding input to motor, including gear loss (watts)	83,900	95,000
Efficiency of motor, excluding gear	93	93·5
Corresponding output from motor to gear (watts)	78,000	88,900
Average internal loss in motor while electricity is on (watts)	5,900	6,100
Time that electricity is on during each run (seconds)	41	44
Total time of running, including 12-second stop (seconds)	110·6	104·4
Ratio of the time that motors are in circuit to the whole running time	0·37	0·42
Average internal loss in watts per motor taken over the whole period	2,180	2,540
Do. watts per ton of motor	870	1,010

The mean value gives 940 watts per ton for the internal losses, which is only some 13 per cent. of the value at the 240-hp rating, and some 17·5 per cent. of the value at the 200-hp rating.

Lancashire and Yorkshire Railway

The data given for the Lancashire and Yorkshire Railway afford two further tests, which are given in Table XXIV. The estimated average outputs from that Table are as below.

Test.	1	2
Average hp output per motor while in circuit during the test runs shown in Chapter V.	93	79

Let us use the same methods as in the previous cases for estimating the motor losses. The motor weighs 2·75 tons, not including gear.

Fig. 43 gives a representative efficiency curve for a motor of 150 hp, and similar to the motor employed on the Lancashire and Yorkshire Railway. The curve is quite sufficient for obtaining approximately correct results for the motor in question. The calculations are set forth in Table LV.

TABLE LV.—CALCULATIONS OF L. & Y. MOTOR LOSSES IN SERVICE.

Test.	1	2
Average output per motor while in circuit (hp)	93	79
Efficiency from representative motor efficiency curve G in Fig. 43, including gear	88·5	87·5
Corresponding input to motor including gear loss (watts)	78,400	67,400
Probable efficiency, excluding gear (from Fig. 43, curve E)	93	93
Corresponding output from motor to gear (watts)	72,900	62,580
Internal loss per motor taken over the time while electricity is on (watts)	5,500	4,720
Ratio of time during which electricity is on to total time (Table XXIV.)	0·400	0·519
Internal loss per motor averaged over the whole running time, including stops (watts)	2,200	2,450
Internal loss per motor in watts per ton of motor	800	890

These results give an average of 845 watts per ton of motor which is 23·2 per cent. of the value for the rated load.

Summarising the results we obtain the values given in Table LVI.—

TABLE LVI.—COMPARISON OF MOTOR LOSSES AT RATED LOAD AND DURING SERVICE.

Type of motor.	Railway on which motor is used.	Weight of motor without gear (tons)	Rated output of motor (hp).	A—Average internal loss at rated load (watts per ton).	B—Average internal loss during service with a standard train (watts per ton).	Percentage $\frac{B}{A}$.	Per cent. which the average load per motor during service with standard train constitutes, of 1-hr rating.
G.E. 66A	Central London	1·75	125	3880	646	16·7	17·3
G.E. 69B	Gt. Northern Piccadilly and Brompton	2·51	200	5390	940	17·5	21·5
			240 (1-hr rating)	7370	940	12·7	17·9
Dick-Kerr	Lancashire and Yorkshire	2·75	150	3640	890	24·5	27·3

The losses of the G.E.66A and the Lancashire and Yorkshire motors at rated load are around the average figure of 4500 watts per ton, which is representative of the internal losses at rated load for railway motors with natural ventilation. The percentage which the continuous rating of railway motors bears to the 1 hour, 75° rating may be anything from 20 to 40 per cent.; and the figure for the totally enclosed type is usually 25 per cent. when it is estimated from the average load carried by the motors during service, as per cent. of the 1-hour rating.

The values for the G.E.69B motor depart from the other values, but this is due to the design which embodies openings in the case, as

already described. This gives a high weight efficiency (*i.e.* a high output in hp per ton of motor). The gear loss at rated load was taken as 4 per cent., and in this case is the same during service, since the curve of Fig. 57 is nearly straight between the values of 100 and 240 hp. The value for the watts loss per ton during service agrees with the other values; while the high figure for the internal loss at rated load necessarily gives a low percentage for the value which the loss during service constitutes of this 1-hour rated load loss.

Single-phase Motors

By the use of forced draught the heating of motors may be considerably reduced for a given weight of motor, or, what amounts to the same thing, a greater loss may be allowed for the same temperature rise than with natural cooling.

With single-phase motors, the efficiency being inherently lower than that of continuous motors, the use of forced draught is much more necessary to obtain motors of reasonable weight for a given output.

Let us estimate the losses in the motors employed on the Heysham line in a similar way to the previous estimations for continuous motors. The output of the Siemens motors used on the Heysham motor-coaches is 180 hp with forced draught, and the efficiency at this output, including gear loss, is seen from Fig. 83 to be 83 per cent.

With this forced draught rating, the input to the motor is—

$$\frac{180}{0{\cdot}83} \times 746 = 161{,}700 \text{ watts,}$$

and the output from the motor axle to the gear is—

$$0{\cdot}87 \times 161{,}700 = 140{,}600 \text{ watts,}$$

the efficiency of the motor, excluding gear, being approximately 87 per cent. at rated load.

Hence the internal losses amount to 21,100 watts or $\left(\frac{21{,}100}{2{\cdot}8} = \right)$ 7540 watts per ton, since the motor weighs 2·8 tons.

This value is considerably above the corresponding figure for motors without forced draught; in fact, a representative value for the watts per ton with forced draught may be taken as some 7000 watts, as against 4500 watts with natural ventilation. These losses correspond to a temperature rise of 75° on the 1-hour basis.

Taking now the Westinghouse motors of the Heysham equipments, they have an efficiency at rated load of 82 per cent.

including gear loss. Thus the input to the motor at the 150 hp forced-draught rating is—

$$\frac{150 \times 746}{0{\cdot}82} = 137{,}000 \text{ watts.}$$

This motor has a gear loss of 5 per cent. at rated load, therefore the output from the motor axle to the gear amounts to—

$$0{\cdot}87 \times 137{,}000 = 119{,}000 \text{ watts.}$$

This leaves 18,000 watts lost in the motor, and as the motor weighs 2·5 tons, the loss works out at—

$$\frac{18{,}000}{2{\cdot}5} = 7200 \text{ watts per ton.}$$

To compare these figures of 7540 and 7200 watts per ton with the corresponding values in continuous motors for the different railways dealt with in this chapter, we can bring together the following values in Table LVII.* :—

TABLE LVII.—COMPARISON OF MOTOR LOSSES FOR SINGLE-PHASE AND CONTINUOUS MOTORS.

Type of electricity.	Motor.	Method of Ventilation.	Output of motor (hp).	Weight of motor (tons).	Average internal losses, in watts per ton at rated load.
Single phase	Siemens-Schuckert	Forced draught	180	2·80	7540
,, ,,	Westinghouse	,, ,,	150	2·50	7200
Continuous electricity	G.E. 66A	Natural	125	1·75	3880
	G.E. 69B	,,	200	2·51	5390
	,,	,,	240	2·51	7370†
	Dick-Kerr	,,	150	2·75	3640

These figures must all be taken as merely indicating that on the average there is a very considerable difference between the loss per ton for natural and forced draught motors. But the precise conditions in each case greatly affect the quantitative results and render strict comparisons very difficult.

* The subject of the heating of railway motors is also dealt with by Armstrong in a paper entitled "A Study of the Heating of Railway Motors," and read before the American Institute of Electrical Engineers, *vide* "Transactions," vol. xix. p. 809.

† The G.E.69B has openings in the case (see p. 159 and Fig. 84 on p. 161).

CHAPTER XIII

THE WEIGHTS AND COSTS OF ELECTRICAL EQUIPMENTS AND OF ELECTRICALLY EQUIPPED TRAINS *

THE weight of an electric passenger train may be divided into four parts—

I. The trucks, including truck frames, wheels and axles, brake rigging, etc.
II. Coach bodies with under-frames, brake cylinders, etc.
III. Electrical equipment, including motors, rheostats, transformers, controllers, collectors, compressor motors, cables, etc.
IV. Passengers.

Although for a given seating capacity, components I. and II. increase in weight slowly with increasing schedule speed and decreasing distance between stops, nevertheless representative weight and cost values may be readily assigned to these two items.

Component III. increases rapidly in weight with increasing schedule speeds and with decreasing distance between stops. The weight of component III. is also very dependent upon the type of electrical equipment. Thus it will be quite different according as the continuous, the single-phase or the three-phase system is employed.

The weight of the electrical equipment for a given schedule speed and a given number of stops per mile will also be dependent upon the continuity of the service. If, after maintaining its schedule for a given time, say one hour, a train remains at rest for half an hour before again resuming its schedule, these intervals of rest enable the electrical apparatus to cool, and consequently the total weight of the electrical equipment may be less than for the case of a train required to maintain uninterruptedly for, say, 15 hours per day, the same

* The first portion of this chapter is on the lines of an article contributed by the author to the *Railway Gazette* of January 22, 1909. The calculations have, however, been completely revised to accord with the more specific and extended data since obtained by the author.

schedule speed with the same number of stops per mile.* In order to arrive at a uniform basis of comparison, let us, in this Chapter, confine our investigation to trains for the latter kind of service, namely, trains running uninterruptedly to their schedule for some 15 consecutive hours per day. For such trains it has been found by experience that the load on the motors, averaged over the entire run, must be of the order of only some 20 per cent. to 25 per cent. of their 1-hour, 75° C. rated capacity.

Component IV. of the total train weight is a very variable factor. While the number of passengers is often, for a short distance, considerably in excess of the seating capacity of a train, the *average* of the number of passengers carried by an urban or suburban train throughout all its journeys is rarely more than 40 per cent. of the seating capacity of the train, and 30 per cent. or, at the most, 35 per cent. would be a much more representative figure.

As will appear later in this chapter, a 180-seat train, equipped with continuous electricity apparatus, and designed for operation at a schedule speed of 26 ml ph with 1 stop per mile, weighs, without passengers, about 88 tons. If one-third of the seats, *i.e.* 60 seats, are occupied by passengers, and if the average weight per passenger is 62 kg, then the aggregate weight of the passengers is—

$$\frac{60 \times 62}{1000} = 3{\cdot}7 \text{ tons.}$$

This brings the weight of the train up to 91·7 tons, an increase of 4 per cent. over the dead weight. Of course, this percentage increase varies considerably with the design of the train as regards the arrangement of seats, but it will, for a given train, rarely average an increase of more than 5 per cent. or 6 per cent. If we take it at 6 per cent. we shall be within a couple of per cent. of the truth for cases when the weight of the passengers for the average conditions of load lies between 4 per cent. and 8 per cent. of the weight of the train, and this will be the case for practically all trains for city and suburban services.

In the following investigation the total train weight will be taken as equal to the dead weight divided by 0·94, *i.e.* multiplied by 1·06. The total train weight is thus made up of the following four components:—

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* On London's tube railways a record of 50,000 miles per annum for a single train is by no means abnormal. Since the speed is of the order of 16 ml ph, this works out at 3100 hours of service per year. But in main-line railways no such record is obtained. Thus, on the Heysham branch of the Midland Railway, although the trains run at a speed of some 30 ml ph the annual mileage per train is only 28,500, which works out at only 950 hours out of the 8760 hours in the year.

I. The trucks.
II. The coach bodies.
III. The electrical equipment.
IV. The passengers.

Let us take the case of a well-built three-coach train, providing 180 seats, with the usual proportion of first-class and third-class seats. Let this train be designed for operation at a schedule speed of 26 ml ph with one stop per mile. If T is the time, in seconds, of a single run from start to stop, and if Q is the duration of each stop in seconds, then—

$$T + Q = \frac{3600}{26} = 138{\cdot}3 \text{ seconds.}$$

Let Q = 20 seconds. Then—

$$T = 138{\cdot}3 - 20 = 118{\cdot}3 \text{ seconds.}$$

The average speed works out at—

$$\frac{138{\cdot}3}{118{\cdot}3} \times 26 = 30{\cdot}4 \text{ ml ph.}$$

From Fig. 46 (page 81), we ascertain that such a train will require an electrical equipment providing 11 rated hp on the 1-hour, 75° C. basis of rating per ton of dead weight of train; but let us be conservative, and provide 12 hp per ton of dead weight of train.

The following rough data of weights and costs will serve the purpose of this investigation:—

Bogie trucks—

Weight of each motor-truck, including truck frames, wheels, axles, and brake rigging .	= 5·5 tons
Weight of each trailing-truck	= 4·0 tons
Cost of trucks	= £22 per ton

Coach bodies—

Weight of each motor-coach body, complete with under-frame, brake cylinders, etc. . .	= 15 tons
Weight of each trailer-coach body . . .	= 11 tons
Cost of coach bodies complete	= £80 per ton

Electrical equipment—

Weight of continuous-electricity equipment	= 19 kg per rated hp
,, of single-phase equipment . .	= 40 kg per rated hp
Cost of electrical equipment . . .	= £125 per ton

Let us work out the weight of,—first, a continuous-electricity train, and, secondly, a single-phase train.

I. Train with Continuous-Electricity Equipment.

I shall make the preliminary assumption that a suitable train for the required capacity and schedule will comprise two motor-coaches and a trailer interposed between them, and that only one of the bogies on each motor-coach will require to carry motors.

Thus we have—

2 motor-bogies at 5·5 tons	= 11 tons
4 trailing-bogies at 4·0 tons	= 16 „
2 motor-coach bodies at 15 tons	= 30 „
1 trailer-coach body at 11 tons	= 11 „
Weight, exclusive of electrical equipment . .	= 68 tons

Let us denote by W the weight of the electrical equipment in tons. An outside figure for modern railway equipments employing continuous motors, each of from 150 to 250-hp rated capacity, is 19 kg per rated hp, and since we require 12 rated hp per ton of dead weight of train (= 68 + W) we have—

$$W = 0{\cdot}019 \times 12 \times (68 + W) = 20 \text{ tons}$$

$$\therefore \text{ Dead weight of train} = 68 + 20 = 88 \text{ tons}$$

Consequently—

the rated capacity of electrical equipment = 88 × 12 = 1056 hp

This may be provided by four 265-hp motors and the auxiliary apparatus.

We may estimate the cost as follows:—

		£
Trucks (= 22 × 27)	=	590
Coach bodies (= 80 × 41)	=	3280
Electrical equipment (= 125 × 20) . . .	=	2500
Labour in assembling (= £6 per ton) . . .	=	530
Total cost of train	=	6900
„ „ per ton	=	£78·5
„ „ per seat	=	£38·3

II. Train with Single-phase Equipment.

We shall require to provide 8 motors—one for each axle. Thus we have—

4 motor-bogies at 5·5 tons	= 22 tons
2 trailer-bogies at 4·0 tons	= 8 „
2 motor-coach bodies at 15 tons	= 30 „
1 trailer-coach body at 11 tons	= 11 „
Weight of train, exclusive of electrical equipment ..	= 71 tons

We may again denote the weight of the electrical equipment by W. Now, single-phase electrical equipment, when the component motors are of 150 to 250-hp rated capacity, have a weight of 40 kg per rated hp. We again require 12 rated hp per ton of dead weight of train (= 71 + W). We now have—

$$W = 0{\cdot}040 \times 12 \times (71 + W) = 66 \text{ tons}$$

$$\therefore \text{ Dead weight of train} = 71 + 66 = 137 \text{ tons}$$

Consequently—

the rated capacity of electrical equipment = 137 × 12 = 1644 hp

This may be provided by eight 210-hp motors and auxiliary apparatus.

The cost works out as follows:—

		£
Trucks (= 22 × 30)	=	660
Coach bodies (= 80 × 41).	=	3280
Electrical equipment (= 125 × 66) . . .	=	8250
Labour in assembling (= £4 per ton) . .	=	550
Total cost of train	=	12,740
,, ,, per ton	=	£93
,, ,, per seat	=	£70·8

Thus for our two 180-seat trains operating a 26 ml ph 1-stop-per-mile service we have the results given in Table LXVIII.

Table LVIII.—Particulars of 180-seat Trains with Continuous and Single-phase Equipments.

	Continuous.	Single-phase.
Dead weight of train in tons	88	137
Total cost of train	£6900	£12,740
Ditto per ton	£78·5	£93
Ditto per seat	£38·3	£70·8
Dead weight of train in tons per seat	0·481	0·761

In order to arrive at a rough estimate of the relative annual costs for capital, depreciation, maintenance and renewals, let us assign 15 per cent. of the cost to cover these factors. These annual costs are thus respectively—

	Continuous electricity.	Single-phase.
Per train	£1035	£1910
Per ton	£11·8	£14·0
Per seat	£5·8	£10·6

Let us next endeavour to estimate a reasonable figure for the number of miles which should be travelled by each train per year. Allowing each train to be in service for 160 days in the year, and

keeping it in operation to its schedule for 15 hours out of each day that it is in service, we find that each train is in service for—

$$160 \times 15 = 2400 \text{ hours per annum.}$$

During this time, at 26 ml ph, such a train will cover—

$$26 \times 2400 = 62{,}400 \text{ miles.}$$

In Fig. 45 (page 80) we see that for this 26 ml ph one-mile schedule, a conservative value for the input to the train from the third rail or overhead conductor will be some 110 w hr per ton-mile.

This figure, however, relates to test runs. In practice, allowing for all the exigencies of everyday service, such as shunting, lighting, making up time of delayed trains, and the various other contingencies of routine service, the consumption may be taken as averaging, per ton-mile of recorded service, a 20 per cent. higher amount, bringing the gross input to 132 w hr per ton-mile. Taking into account that the total train weights, including passengers, are—

For continuous electricity . . .	$1{\cdot}06 \times 88 = 93$ tons
„ single-phase	$1{\cdot}06 \times 137 = 145$ „

and that the corresponding gross inputs are—

	w hr per ton-mile	kw hr per train-mile
Gross input for continuous electricity train .	132	12·3
„ „ „ „ „ .	132	19·1

Let us take the over-all efficiency from the generating station to the train as 80 per cent. for the continuous-electricity system, and 90 per cent. for the single-phase system. Then the outputs from the generating station are as follows:—

For the continuous-electricity system . . . . $= \dfrac{12{\cdot}3}{0{\cdot}80} = 15{\cdot}4$ kw hr per train-mile

For the single-phase system $= \dfrac{19{\cdot}1}{0{\cdot}90} = 21{\cdot}2$ „ „

Since each train covers 62,400 miles per annum, the outputs per train per annum are as follows:—

Continuous electricity	$62{,}400 \times 15{\cdot}4 \times 10^{-6} = 0{\cdot}96$ million kw hr
Single phase . .	$62{,}400 \times 21{\cdot}2 \times 10^{-6} = 1{\cdot}32$ „ „

For a service of this sort, provided there are sufficient trains always in service to ensure some approach to a uniform load, electricity could be purchased from supply companies in many districts at 0·50*d.* per kw hr of high pressure, three-phase electricity as delivered from the generating station, and at 0·55*d.* if supplied from the generating station as high pressure, single-phase electricity. This would not include any costs pertaining to the transmission line from the

generating station to the railway. The cost of electricity per train per annum would thus be—

Continuous electricity . . . $\frac{960{,}000 \times 0{\cdot}50}{240} = £2000$

Single-phase . . . . . $\frac{1{,}320{,}000 \times 0{\cdot}55}{240} = £3020$

The interest, depreciation, and maintenance for a train, and the cost of the electricity for the train, are only two of many large items associated with the total cost of running the train. All the other items may, however, be taken as coming, in the aggregate, to substantially the same total, independently whether the continuous-electricity system or the single-phase system is employed. Thus with the heavy single-phase trains the maintenance and depreciation of the permanent way will be much greater than with the relatively light continuous-electricity trains. The cost of single-phase overhead construction is greater than that of third-rail construction. These two items will fully offset the greater cost of the sub-station machinery in the continuous-electricity system. It is here only proposed to compare the components which we have estimated, as these are the ones including the most essential disparity.

Thus we arrive at values set forth in Table LIX.—

TABLE LIX.—ANNUAL COSTS FOR 180-SEAT TRAINS WITH CONTINUOUS AND SINGLE-PHASE EQUIPMENTS.

	Continuous electricity.	Single-phase.
Dead weight of train	88 tons	137 tons.
A—interest, depreciation, and maintenance of one 180-seat, 26 ml ph, 1 stop-per-mile train, per annum	£1,035	£1,910
B—outlay for electricity for one train per annum	£2,000	£3,020
(A + B)	£3,035	£4,930
Mileage per train per annum	62,400	62,400
(A + B) per train-mile	11·7*d.*	19·0*d.*
,, ,, ton-mile	0·133*d.*	0·139*d.*
,, ,, seat-mile	0·065*d.*	0·105*d.*

Thus the single-phase system costs at least some (19·0 − 11·7 = 7·3) more per train-mile for this particular capacity of train, and for this particular schedule, than the continuous-electricity system costs.

There are, of course, many other expenses associated with train operation, and the total costs of all kinds would usually aggregate, for a 180-seat train, at least some 35*d.* per train-mile, or a matter of from two to three times the above value of (A + B). But since, as already pointed out, A and B are the components chiefly affected by the choice between continuous-electricity and single-phase operation, then, whatever be the precise value of the total costs, the difference

against single-phase will, for a train of this capacity and for this schedule, be a matter of some 7*d.* per train-mile. Thus, if the total cost per train-mile is, for continuous electricity, of the value set forth in the first column of Table LX., then the single-phase cost per train-mile will be of the value set forth in the second column, and the percentage by which the latter exceeds the former cost will be of the value shown in the third column.

TABLE LX.—180-SEAT TRAIN, OPERATING TO A SCHEDULE OF 26 ML PH, WITH ONE 20-SECOND STOP PER MILE, AND AGGREGATING 62,400 MILES SERVICE RUN PER YEAR.

Cost per train-mile.		Percentage greater cost of the single-phase train.
Continuous electricity.	Single phase.	
35*d.*	42*d.*	20
40*d.*	47*d.*	17
45*d.*	52*d.*	15
50*d.*	57*d.*	14

The average fare for urban and suburban railways, taking into account all classes, as also workmen's fares, is around 0·6*d.* per mile. But since a seat is, on the average, occupied for—say, 33 per cent. of its journey—the receipts per seat mile are around—

$$0{\cdot}33 \times 0{\cdot}60 = 0{\cdot}20d.$$

The receipts per train-mile are thus of the order of—

$$180 \times 0{\cdot}20 = 36d.$$

Thus it is evident that the difference between the costs of the two systems is more than sufficient to provide for, or wipe out dividends, were electrification introduced on an extensive scale. Or, looking at the matter from the opposite standpoint, if, as its advocates claim, single-phase can, under these conditions, compete with steam, then the use of the continuous-electricity system would render available for dividends, or for reserve funds, or for expenditure in improving the railway, a further large percentage of the gross receipts.

On the whole, my figure of 0·6*d.* per mile for the average fare errs on the side of being rather high. I have taken it with a view to giving the single-phase system the benefit of any doubt on this score. It is evident from the table that the lower the gross receipts per train-mile, the more unfavourable to the single-phase system are the results of the comparison. For the last six months of the year 1909, the receipts from passengers on the Baker Street and Waterloo Railway, the Great Northern Piccadilly and Brompton Railway, and

the Charing Cross, Euston and Hampstead Railway, averaged 0·189*d.* per seat-mile.

It will be pointed out that the case I have taken, namely, a service in which, with one stop per mile, a schedule speed of 26 miles per hour is maintained, is rather a severe service. I am quite aware of this. But it is the very ability to provide such a service which is often a chief inducement to introduce electric operation. If, still with one stop per mile, we come down to a schedule speed of, say 20, ml ph, while electrification is highly desirable, there are not (except for mountain roads and for tunnels and elevated roads) so strong advantages in its favour as exist in the case I have taken for my example. While at the lower speed (with one stop per mile) the disparity between single-phase and continuous electricity is distinctly diminished, the advantage for continuous electricity is still too great to be overlooked. At the slow schedule of 16 ml ph and one stop per mile, or with any schedule equivalent to this, such as still lower speeds with more frequent stops, or higher speeds and less frequent stops (as on the Midland Railway electrification at Heysham), we come to the range of work where, so far as relates to the rolling stock, it is of much less consequence which system is employed. But for so unattractive a service, there will rarely, with present developments, be found sufficient economic advantage to justify substituting electricity for steam.

A point which has not been sufficiently appreciated is the large percentage which the rolling stock constitutes of the total capital outlay of urban and suburban railways.

Thus take the case of 50 miles of double track, over which trains, each with a seating capacity for 450 passengers, are operated at a headway of 2½ minutes and at a speed of 16 ml ph with 2 stops per mile. For such a service the trains may be taken as accomplishing an aggregate of some 14 million train miles per annum. The distribution of the electrification and rolling stock costs is somewhat as follows—

	£
Generation Station	800,000
Transmission system, including sub-stations .	1,600,000
Continuous-electricity rolling stock . .	3,200,000
Total	5,600,000

The rolling stock constitutes 57 per cent. of the total, and the maintenance and depreciation thereon are enough greater than on the other items, to raise the annual costs associated with the third item to some 75 per cent. of the annual costs associated with the total of the three items.

It is thus evident that a serious disadvantage of the single-phase,

as compared with the continuous-electricity system, is the relatively greater weight and cost of the rolling stock. This disadvantage becomes less the greater the distance between stops and the lower the schedule speed. In other words, we may say that, in respect to seating capacity, the single-phase system is, as regards weight, at a less disadvantage the less the *severity* of the schedule.

In Fig. 85 are given the results of carefully prepared estimates of

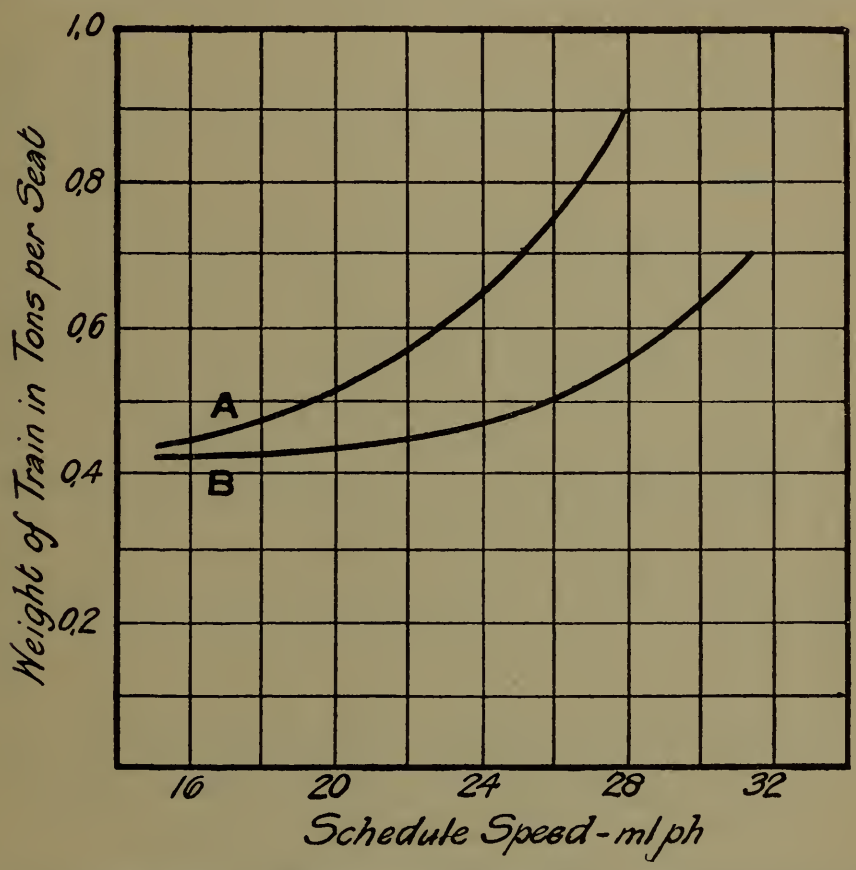

FIG. 85.—Curves showing Weights of a 180-Seat Train for Various Schedule Speeds with One Stop per Mile.
Curve A, for Single-phase Equipment.
„ B, „ Continuous Equipment.

the weights of rolling stock per seat for 180-seat trains designed for operation at various schedule speeds with one stop per mile, and equipped respectively with continuous and single-phase apparatus. We see from the figure that for so light a schedule as 15 ml ph and one stop per mile there is, so far as relates to the weight of the rolling stock, no difference worth considering between continuous and single-phase trains. But already, at 24 ml ph, the single-phase train weighs 35 per cent. more than the continuous-electricity train,

and at 28 ml ph the excess is 60 per cent. For a total train weight of 0·5 ton per seat the continuous train is suitable, with one stop per mile, for a schedule speed of 26 ml ph, as against a schedule speed of only 20 ml ph for the single-phase train.

Corresponding curves of the cost are given in Fig. 86. From these curves we see that for low speeds the cost of the train per seat is of the order of £30 to £35, and that the difference between

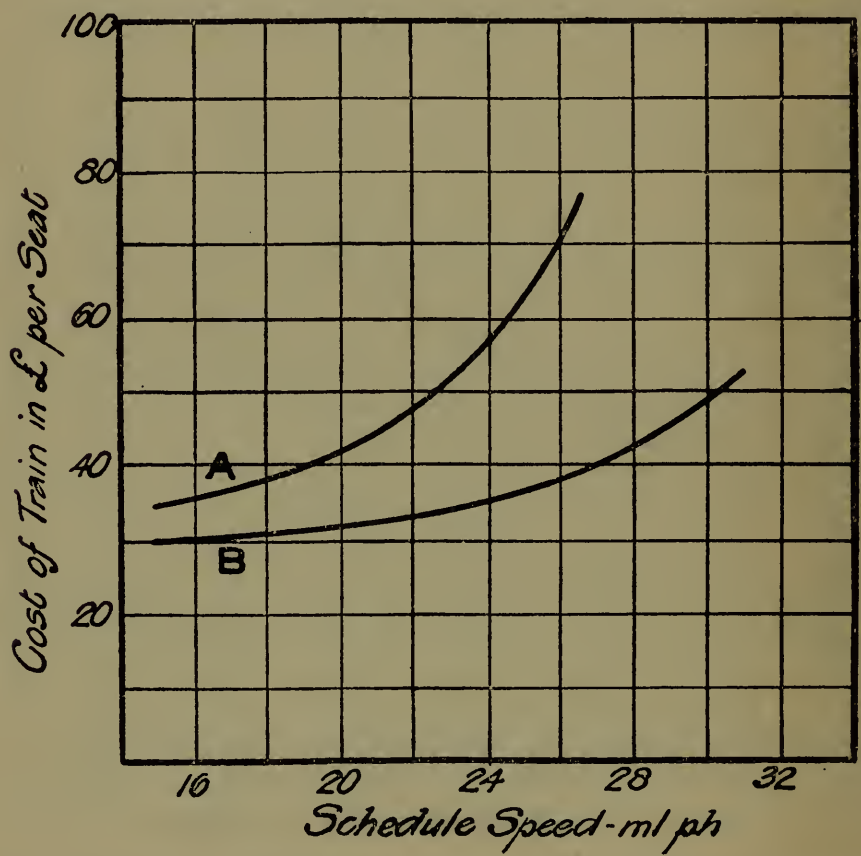

Fig. 86.—Curves showing Costs of a 180-Seat Train for Various Schedule Speeds with One Stop per Mile.
Curve A, for Single-phase Equipment.
,, B, ,, Continuous Equipment.

the two systems, so far as relates to this feature, is slight. With increasing schedule speeds, however, the difference in cost rapidly increases. At some 22 to 23 ml ph (with one stop per mile) the single-phase train costs 50 per cent. more than the continuous-electricity train, and at 27 ml ph the cost of the single-phase train is twice that of the continuous-electricity train. The difference in cost is strikingly brought out in the following table:—

Cost of train per seat.	Appropriate schedule speed in ml ph with one stop per mile.	
	Continuous-electricity train.	Single-phase train.
£40	27	19
£50	30	23

These results show that for the railway electrification work at present confronting engineers, *i.e.* for the range of work where electrical methods are distinctly preferable to steam-locomotive methods, the system employing series-wound, continuous-electricity motors is decidedly the most appropriate.

Extensive areas may be served by the system of traction employing continuous-electricity motors on the trains by the plan, now almost invariably used, of employing large, high-pressure, alternating-electricity generators to provide the electricity in the first instance. The pressure employed at these alternating-electricity generators is usually of the order of from 10,000 to 12,000 volts. The use of these high pressures, enables the Electricity Supply Station to be located at some site selected with reference to economical considerations, and often at a very considerable distance from the trains where the energy is required. Thus a site at the side of a river or canal may, from considerations of the cheapness of land, the facilities for bringing coal, and the plenitude of circulating water, permit of providing electricity at a much lower price than would be possible were the location of the Electricity Supply Station determined solely with reference to its proximity to the location where the electricity is required, namely, at the trains. The very nature of the requirements of railways involves the necessity of supplying electricity over extensive areas, and it is also essential to the economical application of electricity to power supply purposes that there shall be some approach to a uniform load on the station. Many trains must be simultaneously operated from the same Electricity Supply Station in order that their fluctuating individual requirements shall overlap to such an extent as to provide an aggregate load of sufficient uniformity to consist with commercial economy. Thus it has become recognized as fundamental that a large area shall be served from a single Electricity Supply Station. The Lots Road Electricity Supply Station at Chelsea, for example, at the times of maximum load, supplies electricity simultaneously to some 165 trains scattered over an area some 25 miles long and some 10 miles wide. Of course, when a certain distance has been reached, the cost of the copper transmission line becomes sufficiently great to justify a second Electricity Supply

Station from which the trains shall draw the energy necessary for their propulsion when they have passed over from the area supplied by the first station. Usually, however, a single Electricity Supply Station has sufficed, even for very extensive undertakings. Thus the Lots Road Station supplies on the aggregate, some 60 miles of double track and involves, during the times of heaviest traffic, between 200 and 300 trains in service, each train requiring an *average* output from the generating station of some 100 kw.

It is practicable for two independent railways serving adjacent or overlapping areas, and enjoying running powers over one another's lines, to each have its own Electricity Supply Station and to interlink their circuits. In such cases, each railway provides substantially that portion of the total electricity which it requires for its own sections, but there is usually no hard-and-fast demarcation. As an instance of this plan may be cited the fact that the Lots Road Electricity Supply Station of the London Underground Railways Co., and the 6½ miles distant Neasden Eectricity Supply Station of the Metropolitan Railway are thus interlinked. The total length of the line served by the two interlinked systems is some 80 miles of double track. Several other equally extensive systems served by only one or two Electricity Supply Stations are in operation in various parts of the world, and their number is rapidly increasing. Consequently, it may be claimed to have been definitely established that the system of railway electrification employing high-pressure three-phase alternators at the Electricity Supply Station, and carrying 600-volt, series-wound, continuous-electricity motors on the trains for the purpose of their propulsion, is thoroughly appropriate for serving enormous areas in all instances where the density of the traffic is great throughout the area. A fairly dense traffic is essential, since otherwise the overlapping of the demands of the individual trains will not be sufficient to ensure that the apparatus at the Electricity Supply Station is constantly carrying so considerable a percentage of the load for which the station is designed, as to permit of supplying the electricity at a low cost. Thus, it would be absurd to have an Electricity Supply Station whose load consists of only one train equipped with electric motors fed from this station. If the maximum load required by the train should be 1000 hp, then the maximum capacity of the station would also be 1000 hp. But a station to serve 100 such trains would require to have a capacity, *not* of $100 \times 1000 = 100{,}000$ hp, but only, say, some $10 \times 1000 = 10{,}000$ hp, since the 100 trains would not make their maximum demands at the same time. Thus 100,000 hp aggregate capacity of steam locomotives would be replaced by only 10,000 hp aggregate capacity of steam engines in the Electricity Supply Station. And whereas the annual efficiency of each of the 100 small 1000-hp non-condensing locomotives

would only be at the most 3 per cent., the combined annual efficiency of the boiler plant and the four 2500-hp condensing turbines which, in such a case, would be installed in the Electricity Supply Station would be of the order of 12 per cent., and the annual over-all efficiency of the station, from the coal pile to the outgoing cables, would be of the order of 10 per cent. In cases where electrical operation is a sound proposition, the annual outlay for coal at the Electricity Supply Station is far less than would be the outlay for the coal which would be burned in the locomotives which would be required to maintain the same service of trains.* The cost of the electrical equipment brings up the total cost of rolling stock to the rather high figure of some £80 per ton of total weight of train, and it should not be claimed that electrical methods have any advantage as regards the decreased initial cost of a train of a given seating capacity. In fact, the contrary is the case, but owing to the much greater schedule speeds rendered practicable by electrical methods, the annual capital cost of the rolling stock *per train-mile* is brought to a much lower figure than is attained on urban and suburban routes by steam-locomotive trains. A very great advantage also accrues from the fact that, with given terminal facilities, twice as many electrical trains can, as pointed out by Mr. Aspinall,† be dispatched within a given time than is the case with steam trains.

When we come to a consideration of the permanent way, it is now usually agreed that the rails are subject to a decidedly more rapid deterioration on electric than on steam roads. This circumstance, together with the large outlay for the third rail, or overhead construction, brings the annual outlays which must be allocated to structures and apparatus throughout the length of the permanent way, much higher *per mile* with electricity than with steam locomotives. But in view of the high speed and dense traffic rendered practicable with electrical methods, here again the outlays *per train-mile* are less than with steam-locomotive services.

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* On p. 606 of the "Proceedings of the American Institute of Electrical Engineers" for April, 1910, Mr. W. S. Murray states that some tests have been made with 20 steam-locomotives on the New York, New Haven and Hartford Railway, to compare the coal consumption with that for electric trains running under the same conditions. He states that for the density of traffic on that road, they have found that the coal burned at the Electricity Supply Station is only one half as great as that burned with an equivalent steam-locomotive service.

† Mr. Aspinall puts the case as follows: "Every time a locomotive train comes in and goes out, you have four platform operations and eight signal operations. First of all, the train comes in, then a locomotive follows it, that is two; then the train goes out, that is four platform operations, which means eight signal operations. The electric motor train comes in, that is one; the motor-man goes to the other end of the train, and the train goes out, that is two. You have only two platform operations and four signal operations. The result is that, by using motor-car trains instead of locomotives, you double the capacity of your terminal accommodation."

Thus, the fundamental condition for obtaining an adequate return for the heavy costs entailed in "electrifying" a section of steam railway, is that the electric service shall provide very frequent trains. During the hours of densest traffic, some of the London tube railways provide a service, in each direction, of forty trains per hour. This corresponds to one train every 90 seconds. This is in striking contrast to the conditions on extensive sections of main-line railway situated at considerable distances from cities. On such sections, only some couple of trains would pass a given point in one direction in the course of an hour. Obviously, under such circumstances, the proposition to obtain the power from a distant generating station loses force, since the cost of the structures for conveying the electricity to the train works out at a high value per train-mile, and the advantages of the self-contained steam-locomotive are very evident.

If, under these conditions, some autocrat were, nevertheless, to require the supercession of the steam-locomotive by the electric motor, the single-phase system would legitimately come into consideration. From the point of view of the single-phase system, the irony of the situation arises from the circumstance that in the field where its economy is more or less on a par with that of the continuous-electricity system, the steam-locomotive can usually more than hold its own in comparison with electrical methods.

CHAPTER XIV

SUMMARY AND CONCLUSIONS

In a lecture which I delivered in October, 1909, at the School of Military Engineering at Chatham,* I pointed out that in the earlier instances of the application of electric motors to the propulsion of railway trains, the chief motive was usually quite dissociated from any question of the superiority of the electric motor over the steam locomotive as regards capacity or economy. In many instances, the complete elimination of smoke from tunnels was practically the exclusive reason for the adoption of electricity. This same feature of the absence of smoke in tunnels and railway stations, and even on overhead railways, continues to play no small part in the favourable reception which has been accorded to electrically-propelled trains. Owing to difficulties with steam and smoke, the steam locomotive would be altogether inadmissible for hauling trains in the deep-level tubes which now play so important a part in the transportation arrangements of London. Had it not been for this feature of cleanliness a much longer time would have been required to attract to the proposition of railway electrification the very serious attention which it now commands. Amongst the installations where electricity has been adopted primarily to eliminate smoke difficulties may be mentioned the New York Central Railway, the Pennsylvania Railway, and the New York, New Haven and Hartford Railway, which all enter New York through long tunnels, the Baltimore and Ohio Railway, the Simplon Tunnel Railway, the Cascade Tunnel of the Great Northern Railroad, U.S.A., the Mersey Railway, the tube railways of London, the Metropolitan District Railway and the Metropolitan Railway in London, the Berlin Overhead and Underground Railway, the St. Clair Tunnel of the Grand Trunk Railway System, and the Metropolitan Underground Railway of Paris. Unfortunately, on British Railways the restricted dimensions of the tunnels often render them a hindrance rather than an incentive to the introduction of electrical methods. In this respect the conductor

* In this chapter I have made use of certain portions of my Chatham lecture.

rail is more amenable to the ruling conditions than is the overhead conductor system.

It has, however, been for some time clearly recognized that the electrical methods which have already come into fairly extensive use on railways possess inherent attributes which in themselves suffice, quite aside from the first-recognized important feature of greater cleanliness, to ensure their *ultimate* general adoption for a very wide field of railway work now usually done by means of steam locomotives,

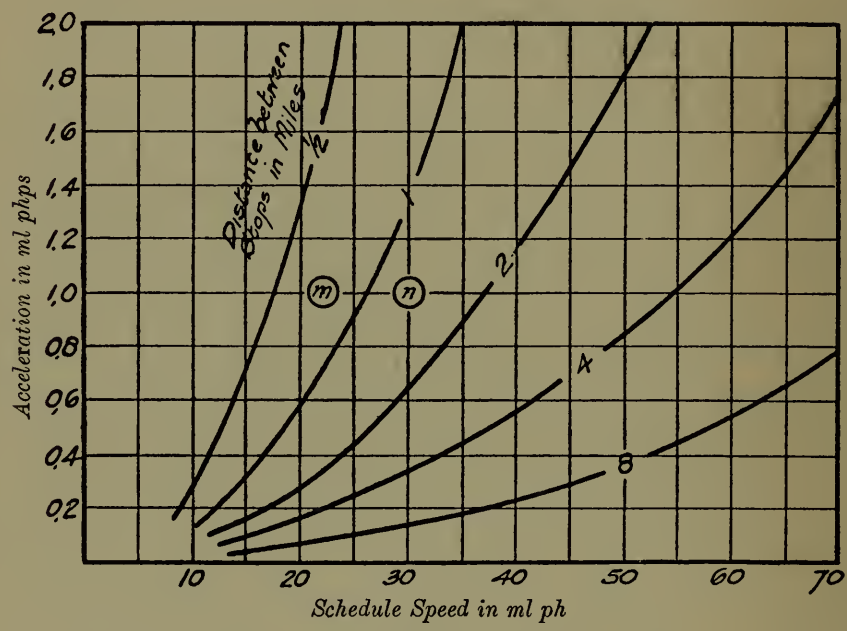

FIG. 87.—Curves showing the Acceleration in ml phps necessary to maintain Various Schedule Speeds for several Distances between Stops.
m = S.L.E. Rly.—Schedule Speed 22 ml ph. Acceleration 1·0 ml phps. Average Distance between Stops 0·88 Mile.
n = L. & Y. Rly.—Schedule Speed 30 ml ph. Acceleration 1·0 ml phps. Average Distance between Stops 1·32 Mile.

provided that no adequately radical improvement in the steam engine is brought forward.

In Fig. 87 is shown, for runs of lengths varying from one-half mile up to eight miles between successive stopping-places, the interdependence which exists between the accelerations and the attainable schedule speeds. In the preparation of these curves I have taken the deceleration during braking as some 1·5 ml phps, and the duration of the stops at stations as 20 seconds. I have taken reasonable characteristics for the speed-time diagrams by means of which the data plotted in Fig. 87 have been derived. From these curves it will be seen that while with one stop per mile, an acceleration of 0·4 ml phps only permits of obtaining a schedule speed of some

17 ml ph we can, by doubling this acceleration, *i.e.* by employing an acceleration of 0·8 ml phps obtain a schedule speed of some 23 ml ph, that is to say, we can increase the schedule speed by 35 per cent. Now, for suburban traffic, the practicability of obtaining high schedule speeds, at the same time providing stops every mile, or even every half-mile, is of enormous commercial importance. With present steam-locomotive practice, where suburban passenger trains are rarely accelerated at more than 0·4 ml phps, the attainment of a schedule speed of 22 ml ph is only practicable when the stopping-places are at least 1·7 mile apart. But with electrically-equipped trains, an average acceleration of 1 ml phps is in accordance with thoroughly established practice, and this ratio permits of operating to a schedule speed of 22 ml ph even when the stops are only 0·8 of a mile apart. Thus, the use of electrically-operated trains for a suburban service permits of having, on a given route, twice as many stops as with trains hauled by steam locomotives, and of nevertheless maintaining the same schedule speed.

Conversely, we may, for the two methods of propulsion, compare the schedule speeds corresponding to a given distance between stops. This leads us to the result that, with a stop every 0·8 mile, the schedule speed with electric trains will be 22 ml ph as against a schedule speed of only some 14 ml ph for trains hauled by steam locomotives. The relation shown in the curves in Fig. 87 as existing between the accelerating rate and the attainable schedule speed is based on the inevitable relations between space and time. The allocation of the high acceleration to electric trains, and of the low acceleration to trains hauled by steam locomotives, is based on experience. As instances of electric railway practice in this country, I may cite the Liverpool-Southport electrified section of the Lancashire and Yorkshire Railway, where trains equipped with series-wound, continuous-electricity motors are regularly operating to the schedule indicated at point *n* in Fig. 87. This point corresponds to a schedule speed of 30 ml ph over a route with 1 stop every 1·32 mile. Point *m* corresponds to the schedule for the single-phase electric trains in operation on the 9-mile section of the L.B. & S.C. Railway between London Bridge and Victoria, and known as the South London Elevated Railway. The average distance between stops on this railway amounts to 0·88 mile and the schedule speed is 22 ml ph. Still higher accelerations have been employed on electric railways and might be cited, but, on the whole, the economical range appears to be of the order of from 1·0 to 1·5 ml phps for high-speed electric trains making frequent stops.*

It is true that steam locomotives may be designed which will be

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* See p. 5, Chapter I.

capable of accelerating trains at a much higher rate than 0·4 ml phps. But just as the economical limit for electrically-equipped trains appears now to have been established by experience to be of the order of from 1·0 to 1·3 ml phps, although twice this rate could readily be provided were it justifiable to go to the necessary expenditure, and were the greater total train weight immaterial; so, also, it appears now to have been established by experience that the economical limit for trains hauled by steam locomotives is, for such a service, of the order of 0·4 ml phps, although there is no reason why twice this rate, or even more, could not be provided were questions of cost and weight not of consequence. Obviously, the acceleration corresponding to a given tractive effort exerted by a locomotive is dependent upon the weight of the train behind the engine. If, with a given weight of train, a certain engine accelerates at 0·3 ml phps it will, with a train of only half this weight, accelerate at 0·6 ml phps.

Let us consider a train operated by continuous electricity, employed on the Liverpool and Southport section of the Lancashire and Yorkshire Railway. It has already been stated that the point *n* of Fig. 87 relates to this train. The train comprises 4 coaches, and its complete weight is 144 tons. The train is made up of 2 motor-coaches, on each of which all four axles are driven, and of 2 intermediate coaches not carrying any propulsion equipment. Thus, out of the train's 16 axles, 8 are driven by motors, and these 8 axles carry 92 tons out of the 144 tons total weight of the train. Owing to the uniform turning effort of electric motors, slipping rarely takes place with electric trains until the tractive effort of the motor amounts to some 25 to 30 per cent. of the weight on the driven axles. Indeed, instances are on record where, when sand has been used, the coefficient of adhesion has amounted to 0·35. Thus, it will be agreed that it is conservative to take the adhesion in the case of electric trains, at 25 per cent. On the basis of 25 per cent. adhesion, we arrive at the result that the wheels will not slip until the 8 motors are exerting a tractive effort of 23 tons. This tractive effort, which would, on a level track, correspond to the high acceleration of 3 ml phps, would call for a current much in excess of that for which the motors and apparatus have been designed, and the calculation has been given simply to illustrate the point that, with electrical traction methods, considerations of adhesion far less often impose limitations than is the case with steam-locomotive methods.

The steam train must carry not only its own motor, the steam engine, but also its steam-raising apparatus and its fuel, and the combined plant cannot, for a given capacity, be reduced to any such small volume and low weight as has become standard for electrical equipments. In electric traction, with specially severe conditions, it is only

the increased weight of the motor and its controlling apparatus which affects the total train weight; the severe conditions as regards the steam-raising apparatus and the weight of the fuel are transferred to the electricity generating station, whereas with trains hauled by steam locomotives any increase in the severity of the service entails increased size and weight, not only of the engine, but of the steam-raising apparatus which is carried on the train. Thus, the weight of trains hauled by steam locomotives more greatly exceeds the weight of electric trains for the corresponding service the greater the severity of the service. The severity of the service is, for good level tracks, chiefly proportioned to the frequency of the stops and the schedule speed. Thus, it is especially in the case of suburban services that electrical methods are superior to steam-locomotive methods as regards the advantages of lesser total train weight. In the earlier chapters of this treatise I have shown that it is exactly for these severe services, namely, for high speeds and frequent stops, that a low train weight per passenger carried, or per seat provided, is of great commercial importance. I have shown this to be a consequence of the most characteristic feature of the mechanical problem of operating trains at high schedule speed notwithstanding frequent stops. This characteristic feature relates to the circumstance that, of the total energy required by the train, a preponderating percentage is used in providing the momentum of the train at its crest speed, and that nearly all this energy of momentum is subsequently wasted as heat at the brake-shoes. I have shown in Chapter III. (see Fig. 21, p. 40), that in the case of a schedule speed of 18 ml ph, with stops every half-mile, the energy stored up in the train as momentum at the instant when the crest speed is attained, amounts to some 55 per cent. of the total energy consumed by the train during the run. With 4 miles between stops, however, even at a schedule speed of 30 ml ph, the percentage stored up as momentum at the instant of crest speed is only some 30 per cent. of the total energy consumed by the train during the run.

For a half-mile run at a schedule speed of 18 ml ph only some 22 per cent. of the energy consumed by the train is usefully expended in its propulsion. But for a 4-mile run, even at 30 ml ph, some 55 per cent. of the total energy consumed by the train is usefully expended in its propulsion. For a service with frequent stops, so small a part of the total energy consumed is available for propulsion, that the total input to the train per ton of weight is inevitably very high, and it becomes of the utmost importance to keep down the cost of the energy consumed by the train by keeping down the weight of the train. This state of affairs should be contrasted with the conditions of long-distance, non-stop runs. For such conditions, where steam-locomotive methods are most appropriate, weight is of decidedly less consequence, since the energy stored up as momentum constitutes an

utterly negligible part of the total energy consumed by the train in making its journey. In such a case, by far the greater part of the energy required by the train is consumed in overcoming the friction of the track and bearings, and the air friction at the ends and sides of the train. But little is gained in such service in reducing the weight of the train.

But in suburban, high-speed, frequent-stop services, the energy required is closely proportional to the weight of the train, since it is largely represented by the momentum of the train at its crest speed. Herein, therefore, we see at once, the chief reason why electrical methods are so much more suitable for high-speed, frequent-stop services than steam-locomotive methods can be.

The transit facilities of London, New York, Berlin, Paris and, in fact, of all the large capitals of the most progressive countries, as well as of the immediate suburbs of these capitals, have now, for some years, comprised magnificent systems of electrically-operated railways. To these instances there are now gradually being added the sections of main-line railways radiating out some twenty miles or more from their city termini. This latter movement is proceeding very gradually. It is retarded by the circumstance of the inherent inappropriateness of electrical methods where the traffic is sparse. Thus, while for some 20 to 25 miles out from London the service of all main-line railways is either already dense, or would rapidly become so under the conditions attending electrification; it would be essential, in the interests of economy *beyond* this 25-mile zone, to change over from electric to steam-locomotive propulsion. The necessity of admitting the superiority of steam-locomotive methods for long-distance, non-stop runs inclines the railway managements to postpone taking up the question, the more especially since there is the further difficulty of justifying in advance, the large capital expenditure incurred in electrifying the termini and the area contained within the 25-mile radius. The suburban traffic of a main-line railway is a very large component of its total business. While the length of route is but a small proportion of the railway's entire length of route, this relatively small length is so intensely utilised that the train-mileage within the suburban zone must constitute a large percentage of the total train-mileage of most of the railways entering London, and it would not be commercially justified to subordinate the interests of this large traffic by inflicting upon it some less appropriate electrical system on the plea that that system is the most appropriate for main-line work. More especially is this the case in view of the fact that the steam locomotive is well adapted to handle all except the suburban sections of a main-line railway.

Largely owing to the importance of employing trains of light weight, it is now generally agreed that of the various leading systems

of electrical propulsion, the system employed on the train, series-wound, continuous-electricity motors is distinctly the best for suburban services with high speeds and frequent stops. The three leading systems of electric railways employ on the trains respectively—

I. Three-phase motors.
II. Continuous motors.
III. Single-phase motors.

The three-phase motor is preferable to either of the other two types so far as relates to its light weight. For the service in question, however, *i.e.* for the particular sort of service where steam-locomotive methods are at the greatest disadvantage, the three-phase motor has the serious disadvantage that it runs at approximately constant speed at all loads. If a three-phase train encounters a grade, it does not ascend at reduced speed as do the other two types of motor, but it maintains its full speed throughout the ascent. This imposes severe peaks of load on the Electricity Supply Stations, whereas the reduced speed at which the other two types of motor automatically operate so soon as the grade is reached, protects the Electricity Supply Station from such wide variations in its load. The capital cost of an Electricity Supply Station for a given quantity of electricity delivered per annum is less the more uniform the load. A railway load is at the best far from uniform, and it is, consequently, the more important from this standpoint to avoid employing a type of motor with properties tending to accentuate the load-fluctuations on the Electricity Supply Station. A lesser disadvantage of the three-phase railway motor, but one which should, nevertheless, be mentioned, is the necessity for providing at least two supply conductors and at least two contact trolleys.

The three-phase system has been very widely employed on the Italian State Railways. Professor Kapp, in his Inaugural Presidential Address to the Institution of Electrical Engineers, on November 11, 1909, quotes Mr. Verola, Chief Engineer of the Electrical Department of the Italian State Railways, as follows:—

"The decision to use the three-phase system is not final and absolute for our administration; but the latter considers it preferable as a beginning, for the lines at present under electrification. The possibility to use the single-phase system in other cases which may better lend themselves to it, is thereby not excluded. In the case of three lines (Pontodecimo-Busalla, Bardonecchia Modane, and Savona-Ceva), which are about to be opened, the service is extremely heavy, trains of 400 tons and over having to be hauled up long grades of 2·5 to 3·5 per cent. at a speed of 45 km ph. With the three-phase system it is possible to comply with these conditions by using two locomotives. These each weigh 60 tons, and each develop (at the

1-hour rating) 2000 hp. They have five driving axles and two motors, which are placed above and between the three middle axles. Connecting-rods transmit the motion from the motor to the driving axles. The three-phase system has the advantage that in running downhill the speed cannot exceed a certain limit, whilst recuperation of energy is possible. With the single-phase system, the weight of the motors would be at least doubled, resulting in a greater expenditure of energy, more especially as we shall be obliged always to use two locomotives to each train. The advantages of wider speed adjustment in running, and better efficiency in starting, are not of importance, since the grades are long and fairly uniform, and the distance between stations is great, whilst the latter are all on the level. For these reasons, and also on account of uniformity in the service, it is probable that also some future electrifications will be on the three-phase system, notably that of the prolongation of the Valtellina line to Milan, which will shortly be taken in hand. It is, however, highly probable that some other lines will be worked single-phase. One of these is the line Turin-Pinerolo-Torre-Pelice, where widely different speeds are necessary, the maximum being 80 km ph for 100-ton passenger trains."

We now come to the second type, namely, the type of motor employing continuous electricity. The complete electrical train equipments employing this type of motor are, on the one hand, as light as equivalent three-phase equipments, and they are far lighter than equivalent single-phase equipments.

Thus, a very conservative figure for the complete weight of electrical equipment when continuous motors are employed, is 19 kg per rated hp as against 40 kg per rated hp for the complete weight of the electrical equipment when single-phase motors are employed.

Let us compare a typical continuous motor with a typical single-phase motor. For the former, we may take the G.E.69B already illustrated in Fig. 84, on p. 161, and for the latter, the W.E.51, of which an illustration is given in Fig. 88. For the commonly-accepted basis of rating of railway motors, namely 75° C. thermometrically-determined rise of temperature after one hour's run at constant load, the continuous motor has a capacity of 240 hp, as against a capacity of only 115 hp for the single-phase motor. The two motors have substantially the same over-all dimensions and weight, and yet the single-phase motor has only half the capacity of the continuous-electricity motor.

It is not alone that single-phase motors are much larger and heavier for a given rated output, but also the remainder of the electrical equipment required on the single-phase system is very much more bulky and heavy than the corresponding apparatus for continuous equipments.

As to the single-phase motor, efforts have been made to reduce its

size by employing a blast of air provided by a ventilating set located at some suitable point on the truck, and from which air is forced to

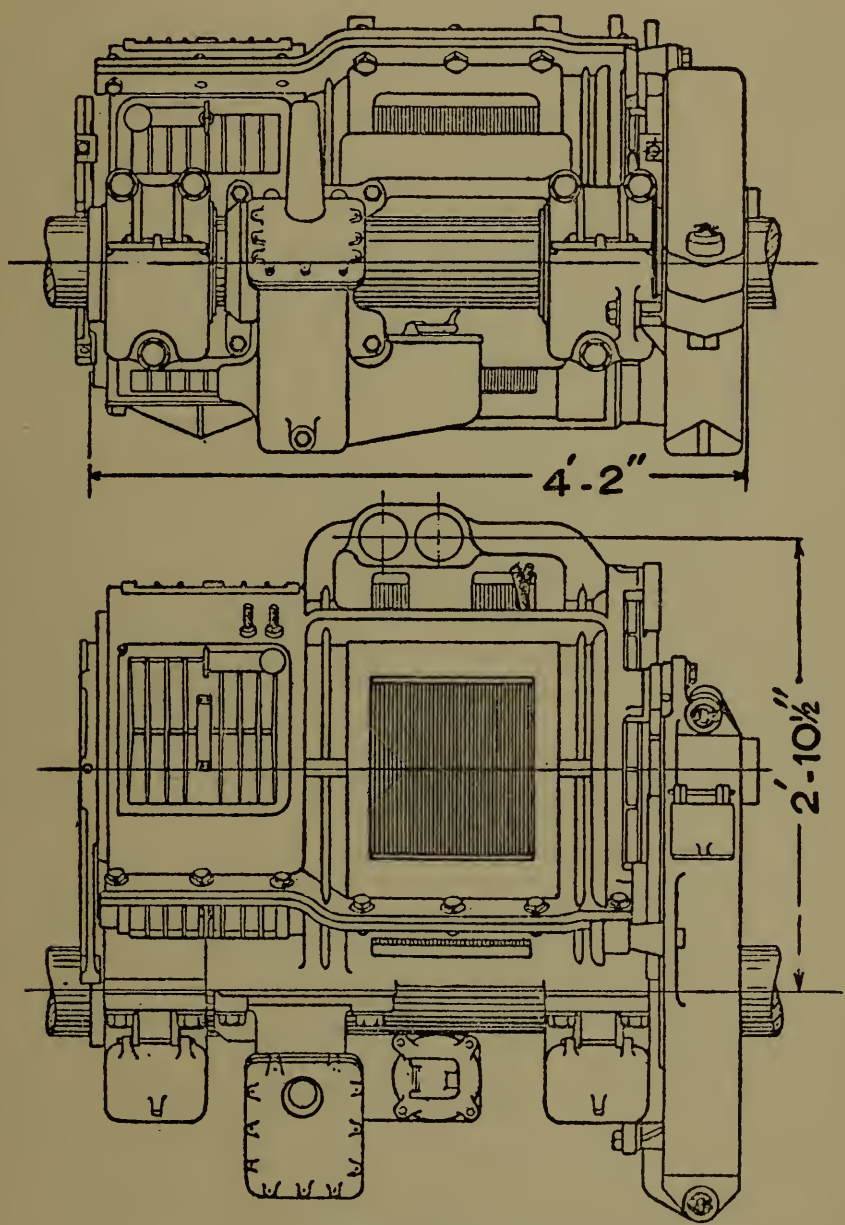

Fig. 88.—W.E.51 Single-phase Motor.

One-hour Rating	115 hp.
Weight (including Gearing) . .	2·75 tons.
Speed at One-hour Rating . .	600 rpm.

and through the motor. By these means, which are employed on the Siemens and the Westinghouse equipments on the single-phase trains

for the Heysham section of the Midland Railway, the motor weight has been brought down to 17·6 kg per hp as against 24 kg per hp for the weight of the single-phase motors on the S.L.E. line. On these S.L.E. motors, the only artificial ventilation is that provided by the small amount of air drawn through a hole in the armature shaft and subsequently escaping from the motor through openings in the case. Mr. Dawson, who has designed the electric trains on the S.L.E. Railway, considers this to be the best plan, notwithstanding the somewhat greater weight of motor. On p. 165 of his book, entitled "Electric Traction on Railways," Mr. Dawson states—

"Such methods (alluding to ventilation by means of fans), although used by most makers of single-phase motors, even in the case of motor cars, are not necessary, nor are they advisable, except where installed on electric locomotives, in the cabs of which plenty of room is available."

The total weights of the electrical equipment on the S.L.E. trains have never (so far as I know) been made public. But assigning to the equipment additional to the motors the average of the weight per hp of the corresponding parts of the Siemens and the Westinghouse equipments on the Heysham section of the Midland Railway, we obtain for these additional equipments, a weight of 21 kg per rated hp capacity of the motor, which, added to the motor-weight of 24 kg per hp, gives 45 kg per hp as the complete weight of the electrical equipments on the S.L.E. trains, whereas for continuous-electricity equipments, the corresponding figure is almost always, in modern designs, well below 20 kg per rated hp of the motor. In Table LXI. I have brought together the leading data of the electrical and mechanical weights of some motor-coaches, which may be taken as typical of the practice of the last few years so far as regards both continuous and single-phase equipments. Certain striking conclusions may be gleaned from this table. Thus, in the cases of the 11 motor-coaches, fitted with continuous-electricity equipments, we find that the average ratio of the "total weight of the electrical equipment per motor-coach" to the "weight of all motors including gearing" is 1·30, the minimum and maximum values of the ratios being respectively 1·26 and 1·33. Thus there is great constancy in this ratio, and it may be taken as typical of continuous-electricity practice. Amongst the S.L.E. data, those marked with an asterisk are based on my own assumptions, since the data have not been published. It is not apparent how any progressive purpose can be served by withholding data of the weights of the equipments on this line.

Numbers 12 to 15 of Table LXI. relate to motor-coaches fitted with single-phase equipments. From these data we see that the ratio of the weight of total electrical equipment to the weight of the

TABLE LXI.—PARTICULARS OF WEIGHTS OF MOTOR COACHES AND ACCESSORIES, USED ON VARIOUS ELECTRIC RAILWAYS.

Type of electricity employed.	Name of Railway.	Designating number of railway.	Electrical equipment. Make and type of motor.	Rated horse-power of motor.	Gear ratio.	Weight of motor without gear (tons).	Weight of motor with gear and gear case (tons).	Number of motors per motor car.	Weight of all motors with gear and gear cases (tons).	Weight of balance of electrical equipment (controllers, contactors, etc.) per motor coach (tons).	Total weight of electrical equipment per motor coach (tons).	Mechanical equipment. Aggregate weight of both trucks, including brake equipment (tons).	Coach body, including underframe, air compressors, seats, upholstery, etc. (tons).	Total weight of motor coach, *i.e.* electrical and mechanical equipment (tons).	Ratio—Weight of total electrical equipment to weight of motors with gears $= \frac{J}{G}$	Weight of motor and gear in kg per horse-power.	Weight of total electrical equipment in kg per horse-power.	Ratio—Weight of total electrical equipment as percentage of total motor coach weight $= \frac{J}{M}$ per cent.
			A	B	C	D	E	F	G	H	J	K	L	M	N	O	P	Q
Continuous.	Central London	1	B.T.-H. G.E. 66A	125	3·94	1·75	1·96	2	3·92	1·00	4·92	5·9	12·2	23·0	1·26	15·9	20·0	21·4
	Gt. Northern and City	2	B.T.-H. G.E. 66	125	4·2	1·75	1·96	2	3·92	1·21	5·13	6·9	10·7	22·7	1·31	15·9	20·9	22·6
	Lancashire and Yorkshire	3	Dick-Kerr	150	1·95	2·75	2·88	4	11·7	3·0	14·7	9·7	22·0	46·4	1·26	19·2	24·2	31·7
		4	Dick-Kerr	125	2·16	1·93	2·07	2	4·14	1·36	5·5	6·0	10·5	22·0	1·33	16·8	22·4	25·0
	Metropolitan (London)	5	Westinghouse	150	3·5	2·20	2·50	4	10·0	2·72	12·7	11·0	14·8	38·5	1·27	16·9	21·5	33·0
		6	B.T.-H. G.E. 69	200	3·3	2·51	2·80	4	5·6	1·68	7·28	10·5	12·7	41·4	1·30	14·2	18·5	24·5
	North Eastern	7	B.T.-H. G.E. 66	125	3·23	1·75	1·96	2	3·92	1·21	5·13	9·66	11·0	29·0	1·31	15·9	20·9	20·0
	Metropolitan District	8	B.T.-H. G.E. 69	200	3·3	2·51	2·80	2	5·60	1·68	7·28	8·5	13·57	29·4	1·30	14·2	18·5	24·8
	Baker Street and Waterloo	9	B.T.-H. G.E. 69	200	3·3	2·51	2·80	2	5·60	1·68	7·28	7·9	13·1	28·3	1·30	14·2	18·5	25·7
	Charing Cross and Hampstead	10	B.T.-H. G.E. 69	200	3·3	2·51	2·80	2	5·60	1·68	7·28	7·9	11·8	27·0	1·30	14·2	18·5	27·0
	Gt. Northern and Piccadilly	11	B.T.-H. G.E. 69	200	3·3	2·51	2·80	2	5·60	1·68	7·28	7·9	12·4	27·6	1·30	14·2	18·5	26·4
Single-phase.	London, Brighton, and S. C. (S.L.E.)	12	A.E.G. W.E. 51	115	4·26	2·46	2·75	4	11·0	9·5*	20·5*	—	—	54	1·86	2·06*	45*	38*
	Midland (Heysham Branch)	13	Siemens	180	2·9	2·8	3·12	2	6·25	8·15	14·4	11·0	15·1	40·5	2·30	17·6	40·7	35·6
		14	British Westinghouse	150	3·6	2·5	2·78	2	5·56	6·04	11·6	11·0	14·9	37·5	2·08	18·8	39·3	31·0
	Rotterdam and Hague	15	Siemens Schuckert	175	about 3·0	2·8	3·0	2	6·0	9·0	15·0	10·0	22·0	47·0	2·50	17·4	43·6	32·0

motors is, for the average of the four equipments, 2·18. Thus it would appear that, for single-phase motor-coaches the weights of the motors must be doubled in order to arrive at the total weight of the electrical equipments. From Table LXI. we see that the weight of the S.L.E. 115-hp motor is 2·75 tons, or 24 kg per hp of rated capacity, as against 17·6 kg per rated hp and 18·8 kg per rated hp respectively for the two types of single-phase motors employed on the Heysham line, and 17·4 kg per rated hp for the Rotterdam-Hague Railway. The average weight of these four single-phase motors is thus 19·5 kg per hp. Averaging the corresponding values for the eleven instances of continuous-electricity equipments, we obtain the value of 15·6 kg per rated hp. The above figures, together with others of an interesting nature which have been deduced from Table LXI., are brought together in Table LXII.—

Table LXII.—Average Values deduced from Data given in Table LXI.

	Motor-coach fitted with continuous equipments.	Motor-coaches fitted with single-phase equipments.	Percentage by which the weights of single-phase apparatus exceeds the weights of continuous equipments.
Ratio of total weight of electrical equipment per motor-coach to weight of motors per motor-coach .	1·30	2·18	—
Weight of motors with gear, in kg per hp	15·6	19·5	25
Weight of total electrical equipment per motor-coach, in kg per hp .	20·2	42·1	108

While for the reasons already given, this great weight of electrical equipment tremendously handicaps the single-phase system as regards its application to suburban services, it does not constitute a disadvantage of such consequence, as compared with the continuous-electricity system, when dealing with long-distance runs with but few stops. But for long-distance runs with but few stops, it is not apparent that electrical methods of propulsion can yet show a net advantage over steam locomotive methods when all the various advantages and disadvantages are subjected to a careful comparison. It is, however, conclusively established that as a means of propulsion for *suburban service*, electrical methods show a great advantage over steam-locomotive methods.

If, as is in my opinion probable, the suburban train mileage constitutes, on the railways of England, a very large or even a preponderating percentage of the total train mileage, then with the further advantages

of the greater cleanliness of electrical methods and the higher speeds, greater train weights and greater grade-ascending capacity which are all rendered practicable by electrical methods, it could ultimately become a sound proposition to also electrify the long-distance, non-stop sections. In that event, which, however, I regard as by no means imminent, the railways would be faced with the proposition of either using continuous electricity because it is the most suitable for the suburban sections, or else of using single-phase electricity because it is the most suitable for long-distance runs with but few stops, or else of using each in its appropriate place and changing over at the limits of a 25-mile radius. The third proposition is in my opinion the correct one, the more especially since the suburban service is best handled by trains of motor-coaches, while the long-distance service is best handled by locomotives, and since, furthermore, the single-phase system is at a less disadvantage for locomotives. Trains despatched from a terminus and bound for a distant locality would be hauled to the boundary of the 25-mile radius by locomotives equipped with single-phase motors, which, as is well known, are operative on circuits supplied with continuous electricity. After crossing the boundary of the 25-mile zone, these same locomotives would draw their supplies from the single-phase circuits with which the line would be equipped beyond the 25-mile zone. As a matter of experience, the continuous-electricity system of railway electrification has been demonstrated to meet, better than any other system, whether steam or electric, all the requirements of a high-speed, frequent-stop service where the trains are run at intervals of but a few minutes apart. For long-distance runs with infrequent trains, the single-phase system shows up to better advantage, but both are, in such a case, distinctly inferior to a steam-locomotive system.

It must not be assumed that the series-wound, continuous-electricity motor has yet reached the limits of its development. On the contrary, there are various directions in which it may be still further perfected. Its weight may be slightly decreased by the device of forcing air through it, in a manner analogous to that often employed with single-phase motors, as, for instance, with the Siemens and the Westinghouse motors on the Heysham section of the Midland Railway. By forced ventilation the rating of a motor may readily be increased by 25 per cent., and it is probable that still greater improvement in this direction will be found thoroughly practicable after a little more experience has been gained in this direction. It may certainly be considered as fully established that the weight of continuous-electricity motors may be decreased by some 20 per cent. by means of employing forced ventilation, as is usually done with single-phase equipments. For schedules of only moderate severity

there is not sufficient occasion to resort to such methods for the continuous system, since the total weight of the electrical equipment is not then a large percentage of the total train weight. But the adoption of this method should be valuable for especially severe schedules where, even with continuous equipment, the total weight of the electrical equipment constitutes quite a large percentage of the total train weight. As showing the general order of magnitude of the percentage which the total weight of electrical equipment constitutes of the total train weight with continuous equipments, I have prepared Table LXIII.—

TABLE LXIII.

Schedule speed (ml ph).	Distance between stops (miles).	Weight of electrical equipment as a percentage of the total train weight.
16	0·5	10
25	1·0	20
35	1·5	30
43	2·0	40

In corresponding schedules with single-phase equipments, these, where commercially possible, entail much greater weights of electrical equipment. Thus, on the South London Elevated Railway, although the weight of the complete electrical equipment has not been published, it doubtless constitutes at least 41 tons out of the 138 tons of the total train weight, and is thus at least 29 per cent. of the total weight of the train, although the schedule speed is only 22 ml ph, with stops every 0·88 mile.

Equipments comprising continuous-electricity motors are thus so reasonably light that it is legitimate to consider improving their quality even at the expense of slight increase in weight. It is with this thought that several companies are putting on the market continuous-electricity motors with interpoles. The Siemens-Schuckert Co. has supplied motors of this type for the Bonn-Cologne Railway which have the additional feature of being wound for 1000-volt continuous electricity. By means of this feature of interpoles, the obtaining of good commutation is so greatly simplified as to render the use of much higher pressures at the motor a thoroughly sound proposition. By substituting a pressure of 1200 volts for the now customary 600 volts, the sub-stations can be placed at least twice as far apart. This will further decrease the capital and operating costs, and will give a much more uniform load on the sub-station,

since the load at any instant will be made up of the average of twice as many trains.*

Manufacturers are at present making rapid progress toward the use of 1200 volts. Several inter-urban lines are at present either being installed or in operation, providing electricity to the coach or train at a pressure of at least 1200 volts. But in almost all instances, two 600-volt motors insulated for 1200 volts are connected in series across the 1200 volts. In Table LXIV. is given a list of 11 roads where 1200-volt continuous equipments are either being supplied or are already in operation. Amongst the above roads, Nos. VIII. and X. are of especial interest. These two roads were originally equipped with the single-phase system; but the system proved unsatisfactory, and is being replaced by the high-pressure continuous-electricity equipments indicated in Table LXIV. The engineers of the Pennsylvania Railway made some very thorough tests with a single-phase locomotive on a special testing track. The single-phase system was, however, found so distinctly inferior to the continuous system that the railway is obtaining 24 continuous-electricity locomotives to haul its trains through the tunnel approaches to New York City. Each of these 24 locomotives is of 4000-hp rated capacity, and is guaranteed to haul 500-ton trains at a speed of 70 ml ph. Each locomotive is of the articulated type, weighs 150 tons, and is equipped with two 2000-hp motors, the weight of each motor being 19 tons. These are the most powerful *electric* locomotives yet built. The Seebach-Wettingen line of the Swiss State Railways was operated by the single-phase system for several years; but the system was found to cost more than would be the case with steam-hauled trains, and the single-phase system is now being replaced by steam locomotives.

These numerous set-backs are in striking contrast to the almost uniformly successful history of the continuous-electricity system of railway electrification. While the system as it stands is highly satisfactory, it may be of interest to mention certain other directions in which manufacturers are endeavouring to further develop it. Thus in a new line of railway motors with interpoles, advantage is taken

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* An interesting paper giving comparisons between 1200-volt and 600-volt continuous-electricity systems for various services has been read by C. E. Eveleth before the American Institute of Electrical Engineers (see "Transactions," 1910, vol. xxix. pt. 1).

The present author contributed an article to the *Electrical Review* (1904, vol. liv. pp. 693 and 765) entitled "The Continuous System and the Single-Phase System for Traction," in which comparisons were made between a 1300-volt continuous system with two standard motors in series, and a 3000-volt single-phase system. The article evoked correspondence (*Electrical Review*, vol. liv. p. 1031) which is also of interest as bearing so closely on a subject to which more attention is now being given. The method there advocated by the present author is that which is employed on the roads of which data is given in Table LXIV.

TABLE LXIV.—PARTICULARS OF ELEVEN RAILWAYS EQUIPPED WITH THE 1200-VOLT CONTINUOUS-ELECTRICITY SYSTEM.

Designation.	Name of railway.	Total mileage.	Motor-coach equipments.					Sub-station units.		
			Number of motor-coaches equipped.	Number of motors per motor-coach.	Capacity of each motor (hp).	Aggregate capacity of motors per motor-coach (hp).	Total capacity of motors installed. (Rated hp 1 hr 75°).	Number and type of units. R = rotary converter. MG = motor generator.	Capacity of each unit (kw).	Total capacity installed in sub-stations (kw).
I.	Indianapolis and Louisville Traction Co., Louisville, Ky.	41	10	4	75	300	3,000	*	—	—
II.	Pittsburg, Harmony, Butler and New Castle, Pittsburg, Pa.	64	16	4	75	300	4,800	7MG	400	2,800
III.	Southern Cambria, Johnstown, Pa.	24	4	4	75	300	1,200	*	—	—
IV.	Sapulpa Interurban, Sapulpa, Okla.	9	7	4	50	200	1,400	*	—	—
V.	Indianapolis, Columbus, and Southern Traction Co., Columbus, Ind.	41	3	4	75	300	900	*	—	—
VI.	Aroostook Valley, Presque Isle, Maine	12	2	4	50	200	400	2R	200	400
			1	4	75	300	300			
VII.	Central California Traction Co., Stockton, Cal.	16	6	4	75	300	1,800	2MG	500	1,000
VIII.	Washington, Baltimore and Annapolis Electric, Baltimore, Maryland	60	30	4	75	300	9,000	15R	300	4,500
			3	4	125	500	1,500			
IX.	Southern Pacific, Oakland, Cal.	12	44	4	125	500	22,000	12R	750	9,000
X.	Milwaukee Electric, Milwaukee, Wis.	90	15	4	125	500	7,500	12R	300	3,600
			15	4	75	300	4,500	7R	500	3,500
XI.	Shore Line Electric, New Haven, Conn.	52	12	4	50	200	2,400	6R	200	1,200
	Totals	421	168	—	—	—	60,700	—	—	26,000

* Entire line supplied direct from power station.

of the better commutation of such motors to introduce a more efficient method of control, namely, to regulate the field strength by varying resistance in parallel with the series winding. By adjusting the value of this resistance, the speed of the motor may be varied. It would appear probable that a distinct reduction in the energy required at the train should be effected by this method as compared with the ordinary speed-control method by means of resistance in series with the motor. Similar methods of control were often used many years ago on ordinary traction motors without interpoles, but were abandoned owing to the poor commutation performance of the motors when running with a shunted field. The reversion to this method of control, now that improved commutation by the use of interpoles is available, is a thoroughly rational proposition, and bids fair to be attended with success.

Other interesting propositions relate to the employment of shunt-wound, continuous-electricity motors. Such motors can be made to act as generators by simply increasing the field strength above that corresponding to the speed at which the motor is running. This sends electricity back into the line. In other words, a large part of the momentum which, as we have seen, often constitutes a preponderating percentage of the total energy sent into the train, may by this means be returned to the line instead of being wasted at the brake-shoes. A leading disadvantage is that since the motor serves not only to propel the car but also as a generator during braking, it is carrying current for nearly all the time, and consequently must be larger and heavier in order not to overheat. Consequently, the development of a rational regenerative control system should include the feature of forced ventilation of the motors. In addition to the very considerable saving in energy rendered practicable by regenerative control, there is a large saving due to the elimination of the wear between brake-shoes and wheels which, in ordinary equipments, entails heavy annual outlays for renewals.

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